Use Gaussian Quadrature with “enough” sample points to integrate the following functions to a satisfactory accuracy:

\[ I_1 = \int_0^4 e^{-x} \, dx = 0.98168436, \]
\[ I_2 = \int_{-3}^3 e^{-\frac{1}{2}x^2} \, dx = 2.4998609. \]

Your program should include:

(a) A prompt for the function as the interactive input and the interval of integration and the number of sample points, \( M \), to be used.

(b) A subprogram to generate the Gaussian Quadrature Abscissas and Weighting factors. The abscissas are the roots of the \( M \)-th order Legendre Polynomial which could be generated using the recurrence relationship:

\[ P_{n+1}(x) = \frac{(2n + 1)x}{n+1}P_n(x) - \frac{n}{n+1}P_{n-1}(x) \]

and the starting polynomials \( P_0(x) = 1 \) and \( P_1(x) = x \). The weighting factor, \( w_i \), can be determined using its corresponding abscissa value as

\[ w_i = \frac{2(1 - x_i^2)}{(n+1)^2[P_{n+1}(x_i)]^2} \]

(c) Since the standard Gaussian Quadrature is derived for the interval \([-1, 1]\), use a change of variable to perform the integral using

\[ \int_a^b f(x) \, dx \approx \frac{b-a}{2} \sum_{i=1}^M w_i f \left( \frac{b-a}{2} x_i + \frac{b+a}{2} \right). \]

(d) Experiment with \( M \) to get accurate results for the integrals. For your reference, using Simpson’s Rule it required 65 sample points for \( I_1 \) and 127 sample points for \( I_2 \). Gaussian Quadrature should require about half as many. Keep in mind that matlab accuracy will break down when \( M \) is greater 40 or so. Mathematica has the capability to do higher order formulas.

Email the diary file which includes the main program and functions and runstream to ce108@usc.edu.