1) Find the integral \( \int e^{3x} \cos(2x) \, dx \).

2) Find the integral \( \int \tan^{-1}(1 + x) \, dx \).

3) Find the derivative of
\[
\frac{d}{dx} \left( \frac{y \cos^3 x}{1 + \sin^2 x} \right).
\]

4) The Gregory Coefficients, \( C_n \), are used to expand the reciprocal logarithmic function,
\[
\frac{z}{\ln(1 - z)} = \sum_{n=1}^{\infty} C_n z^n ; \quad |z| < 1 .
\]

It is given that \( C_0 = -1 \), and the later coefficients can be determined by the recurrence relationship
\[
C_n = -\sum_{k=0}^{n-1} \frac{C_k}{n + 1 - k}.
\]

Find the coefficients in fractional form, \( C_i \), \( i = 1 \) to 12. (Hint: \( C_1 = \frac{1}{2}, C_2 = \frac{1}{12}, C_3 = \frac{1}{24}, C_4 = \frac{19}{720}, C_5 = \frac{9}{100}, C_6 = \frac{863}{60480} \).)

5) Write \( \pi \) to 40 places.

6) Write \( e \) to 45 places.

7) Write the eleven binomial coefficient for \( \binom{10}{0} \) to \( \binom{10}{10} \).
(8) Given $P_0(x) = 1$ and $P_1(x) = x$, the higher order Legendre Polynomials can be obtained by the relationship,

$$P_{n+1}(x) = \frac{(2n + 1)x}{(n + 1)} P_n(x) - \frac{n}{(n + 1)} P_{n-1}(x)$$

For example,

let $n = 1$ : \[ P_2(x) = \frac{3}{2} x(x) - \frac{1}{2} (1) = \frac{1}{2} (3x^2 - 1) \]

let $n = 2$ : \[ P_3(x) = \frac{5}{3} x \left( \frac{1}{2} \right) (3x^2 - 1) - \frac{2}{3} (x) = \frac{1}{2} (5x^3 - 3x) \]

Find all Legendre Polynomials from $P_4(x)$ to $P_6(x)$.

(9) Find the Fourier Sine Expansion of a function $f(x)$ in the form,

$$f(x) = \sum_{n=1}^{N} a_n \sin(n\pi x),$$

in which the Fourier Coefficient for the interval $x=[0,1]$ is evaluated as

$$a_n = 2 \int_{0}^{1} f(x) \sin(n\pi x) \, dx.$$ 

Expand $f(x) = x(x^2 + x - 2)$ in a Fourier Sine Series.

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<th>$x$</th>
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