Generation of Ground Vibrations by Superfast Trains

Victor V. Krylov

Centre for Research into the Built Environment, Nottingham Trent University,
Burton Street, Nottingham, UK, NG1 4BU

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ABSTRACT

Ground vibrations generated by superfast trains are studied theoretically, taking into account the contribution of each sleeper of the track subjected to the action of the carriage wheel axles. It is shown that a very large increase in vibration level may occur if the train speed exceeds the velocity of Rayleigh surface waves in the ground, a situation which might arise, for example, with TGV passenger trains for which speeds over 500 km/h have been achieved on the experimental track in France. The results are illustrated by numerically calculated graphs of spatial distributions and frequency spectra of ground vibrations generated by trains moving with different speeds. Simple mitigation measures based on waveguide effects for ground vibrations are suggested.

1 INTRODUCTION

Railway-induced ground vibrations are one of the major sources of noise and vibration pollution in urban areas.\textsuperscript{1-4} The theory of ground vibration generation by moving trains due to the quasistatic pressure of wheel axles onto the track–soil system has recently been developed.\textsuperscript{5,6} In the case of welded rails and perfect wheels, this mechanism is the major contributor to the trainspeed-dependent components of the low-frequency vibration spectrum (up to 50 Hz).

In the present paper, the above theory is extended to consider excitation of ground vibrations by superfast trains, i.e. trains moving with speeds close to or greater than 300 km/h.
The continuous increase of train speeds on European railways makes it desirable to consider the environmental impact of such trains. From the point of view of generating ground vibrations, it is particularly important to study the effect of trains approaching the 'sound barrier' with regard to the velocity of Rayleigh surface waves propagating in the ground. In May 1990, nine runs of TGV trains at over 500 km/h (i.e. over 138.8 m/s) were made by the French Railway Company (SNCF) on the section of the track between Courtalain and Tours. These speeds are greater than the velocities of Rayleigh waves in soft sandy soils (90–130 m/s). Hence, significant radiation effects for ground vibrations might be expected in these areas, similar to those of Mach radiation of shock waves by supersonic jets or Cherenkov radiation of light by electrons moving with speeds exceeding the velocities of light in the media.

In the following sections, we briefly review the general theory necessary for understanding the generation of ground vibrations by superfast trains. Then we consider in more detail the case of 'trans-Rayleigh trains', i.e. trains moving with speeds greater than the velocities of Rayleigh surface waves in the ground. In the last section we give the results of numerical calculations and discuss some physical aspects of the above phenomena. Finally, some practical conclusions are given and simple mitigation methods suggested based on waveguide effects.

2 OUTLINE OF THE THEORY

In what follows we will consider a train moving with speed $v$ on welded track with sleeper periodicity $d$. The quasistatic pressure mechanism of excitation results from load forces applied to the track from each wheel axle, causing downward deflection of the track. These deflections produce a wave-like motion along the track with speed $v$ that results in a distribution of each axle load over all the sleepers involved in the deflection distance. Each sleeper, in turn, acts as a vertical force applied to the ground during the time necessary for a deflection curve to pass through the sleeper. This results in generation of elastic ground vibrations. Since, in the low-frequency band, the characteristic wavelengths of generated elastic waves are much larger than the sleeper dimensions, each sleeper can be considered as a point source. Calculating the vibration field radiated by a moving train requires the superposition of fields generated by each sleeper activated by all axles of all carriages, with the time and space differences between sources (sleepers) being taken into account.

Using the Green's function formalism, the general expression for the vertical component of the railway-induced ground vibration velocity
spectra on the ground surface \((z = 0)\) may be written in the form\(^{5,6}\)

\[
v_z(x, y, \omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(x', y', \omega) G_{zz}(\rho, \omega) \, dx' \, dy'
\]  

(1)

where \(x\) is the coordinate along the track, \(y\) is the coordinate perpendicular to the track, \(G_{zz}(\rho, \omega)\) is the corresponding component of the elastic Green's tensor (Green's function) for the elastic semispace (we take into account only the Rayleigh surface wave contributions to \(G_{zz}\) since they carry most of the generated energy), \(\rho = [(y' - y)^2 + (x' - x)^2]^{1/2}\) is the distance between the point of observation \((x, y)\) and the elementary source \((x', y')\), \(\omega = 2\pi F\) is the circular frequency, and \(P(x', y', \omega)\) describes the total distribution of load forces along the track. This distribution is proportional to the Fourier transform of the total time- and space-dependent track deflection function. For the light passenger trains considered here, the track deflection function for one axle load \(w(x)\) has the 'classical' form\(^{7,8}\)

\[
w(x) = \left(\frac{T}{8EI\beta^3}\right) \exp \left(-\beta|x|\right) \left[\cos \left(\beta x\right) + \sin \left(\beta|x|\right)\right] + \frac{p}{\alpha}
\]

(2)

where \(T\) is the axle load applied at \(x = 0\), \(E\) and \(I\) are Young's modulus and the cross-sectional momentum of the beam, \(\beta = (\alpha/4EI)^{1/4}\), \(\alpha\) is the proportionality coefficient of the elastic foundation which depends on the stiffness of the ground and of the rubber pads inserted between rail and sleepers,\(^9\) and \(p\) is the uniform weight of the track. The value of the characteristic deflection distance \(x_0\) is determined from eqn (2) as \(x_0 = \pi/\beta\). Note that eqn (2) is valid for the axle loads \(T \leq T_{cr} = (2p/\beta) \exp \left(\pi/\beta\right)\). For \(T > T_{cr}\) the solution becomes more complicated and involves peripheral bulges of the track with loss of contact between track and soil. The values of \(T_{cr}\) are usually not achieved by passenger trains in Europe. For example, for typical parameters of British Rail tracks \((p = 3000 \text{ N/m}, \beta = 1.28 \text{ m}^{-1})\) \(T_{cr}\) is 108 kN\(^{5,6}\), i.e. far above the axle loads of passenger trains.

The time dependence of force applied to the ground from a sleeper located at \(x = 0\) may be determined by the formula

\[
P(t) = T \left[2w(vt)/w_{\text{max}}\right](d/x_0)
\]

(3)

where \(w_{\text{max}}\) is the maximum value of \(w(vt)\). Terms on the right of \(T\) take into account the distribution of axle load between sleepers within the deflection distance. To derive eqn (3), one should use the force balance equation to determine the effective number of sleepers \(N_{\text{eff}}\) equalising the applied axle load \(T\):

\[
\sum_{m=-N_{\text{eff}}/2}^{N_{\text{eff}}/2} \frac{T \, w(md)}{N_{\text{eff}} \, w_{\text{max}}} = T
\]

Here \(m\) denotes a number of a current sleeper. Numerical solution of the
above equation shows that for $\beta$ within the range of interest (from 0.2 m$^{-1}$ to 1.3 m$^{-1}$) the value of $N_{\text{eff}}$ may be approximated by a simple analytical formula $N_{\text{eff}} = \pi/2\beta d = x_d/2d$ which gives eqn (3) after replacing in $w(x)$ the argument $x$ by $vt$.

The Fourier transform of $P(t)$: $P(\omega) = (1/2\pi) \int_{-\infty}^{\infty} P(t) \exp(i\omega t) dt$, taking into account eqn (2), has the form

$$P(\omega) = \left( Td/\pi x_0 \right) \left\{ (2\beta v + \omega)/[(\beta v)^2 + (\beta v + \omega)^2] 
+ (2\beta v - \omega)/[(\beta v)^2 + (\beta v - \omega)^2] \right\}$$

(4)

For a single axle load moving with speed $v$ along the track on perfectly elastic ground the load force distribution may be written in the form

$$P(t, x', y' = 0) = \sum_{m=-\infty}^{\infty} P(t - x'/v) \delta(x' - md) \delta(y')$$

(5)

where $m$ is a number of a current sleeper, $P(t-x'/v)$ describes the time dependence of force applied to the ground from a sleeper with coordinate $x = x'$, and Dirac's delta-function $\delta(x'-md)$ takes the discreteness and periodic distribution of sleepers into account. Taking a Fourier transform of eqn (5) and substituting it into eqn (1) yields the expression for the vertical surface vibration velocity of Rayleigh waves generated at the point of observation $\{x, y\}$ by a single axle load:

$$v_z(x, y, \omega) = V(\omega) \times \sum_{m=-\infty}^{\infty} \exp \left\{ i(\omega/v) md + i(\omega/c_R) \rho_m / \sqrt{\rho_m} \right\}$$

(6)

Here $\rho_m = [y^2 + (x - md)^2]^{1/2}$ is the distance between the sleeper labelled 'm' and the point of observation, and $V(\omega)$ is the function proportional to the Fourier transform of $P(t)$ and to the elastic parameters of the ground:  

$$V(\omega) = (\pi/2)^{1/2} P(\omega) (-i\omega) q(k_R)^{1/2} k_2 \exp \left\{ -i3\pi/4 \right\}/\mu F'(k_R)$$

(7)

where $k_R = \omega/c_R$ is the wave number of a Rayleigh surface wave in the ground, $c_R$ is the Rayleigh wave propagation velocity, $k_1 = \omega/c_1$ and $k_t = \omega/c_t$ are the wave numbers of longitudinal and shear bulk elastic waves, $c_1 = [(\lambda + 2\mu)/\rho_0]^{1/2}$ and $c_t = (\mu/\rho_0)^{1/2}$ are longitudinal and shear propagation velocities, $\lambda$ and $\mu$ are Lamé constants, $\rho_0$ is the ground mass density, and $q = (k_R^2 - k_t^2)^{1/2}$. The factor $F'(k_R)$ is a derivative of the Rayleigh determinant $F(k) = (2k_2^2 - k_1^2)^2 - 4k^2(k_2^2 - k_1^2)^{1/2}(k_2^2 - k_t^2)^{1/2}$ versus $k$ taken for $k = k_R$.

It follows from eqn (6) that a single moving load generates a quasi-discrete spectrum with frequency peaks close to $f_p s$, where $f_p = v/d$ is the so-called passage frequency ($s = 1, 2, 3...$). Deviation from perfect discreteness results from the $i(\omega/c_R) \rho_m$ term in eqn (6) which takes into
account pathlength differences of waves propagated from each sleeper to the point of observation.

Taking account of all axles and carriages of a moving train results in the more complicated formula for the load force distribution over the ground

\[ P(t, x', y' = 0) = \sum_{m=-\infty}^{\infty} \sum_{n=0}^{N-1} A_n [P(t - (x' + nL)/\nu) + P(t - (x' + M + nL)/\nu)] \delta(x' - md) \delta(y') \]

Substitution of eqn (8) into eqn (1) gives the following expression for the frequency spectra of vertical vibrations at \( z = 0 \):

\[ \nu_z(x, y, \omega) = V(\omega) \sum_{m=-\infty}^{\infty} \sum_{n=0}^{N-1} A_n \exp \left( -\frac{\gamma \omega \rho_m}{c_R} \sqrt{\rho_m} \right) \]
\[ \times [1 + \exp(iM\omega/\nu)] \exp \left( i((\omega/\nu)(md + nL) + i(\omega/c_R)\rho_m) \right) \]

Here \( N \) is the number of carriages, \( M \) is the distance between bogies in each carriage and \( L \) is the total carriage length (note that \( M, \ L \) as well as \( d \) may take account of random deviations from their mean values). The dimensionless quantity \( A_n \) is an amplitude weight factor to account for different carriage masses. In writing eqn (9) we account for attenuation in soil by replacing \( 1/c_R \) in the exponentials by the complex value \( 1/c_R + i\gamma/c_R, \) where \( \gamma \ll 1 \) is a constant describing the 'strength' of dissipation of Rayleigh waves in soil, which is assumed to be linearly dependent on frequency.\(^{12}\)

It follows from eqn (9) that spectra of train-induced ground vibrations are quasi-discrete, with the maxima at frequencies determined by the condition \( (\omega/\nu)(md + nL) = 2\pi l \), where \( l = 1, 2, 3, \ldots \). Obviously, \( n = 0 \) corresponds to the passage frequencies \( f_p \) determined by the sleeper period \( d \). Other more frequent maxima are determined either by the carriage length \( L (m = 0) \) or by a combination of both parameters (for \( n \neq 0, m \neq 0 \)). Note that spectra calculated according to eqn (9) are in satisfactory agreement with experimentally observed spectra.\(^{6}\)

3 GROUND VIBRATIONS FROM SUPERFAST TRAINS

The theory described in the previous section is applicable to trains moving with arbitrary speeds. However, for the specific case of 'trans-Rayleigh trains' an additional analytical treatment is useful to elucidate the special features of the problem and to clarify the time and space distributions of radiated waves.
We first consider the vibration field generated by a single load moving with speed \( v \) along a part of a track having a small number of sleepers \( 2Q + 1 \). Let the point of observation be arbitrarily located on the ground surface, i.e., \( \rho_m = [v^2 + (x - md)^2]^{1/2} \). Then, for far-field distances \( R \gg \delta \), where \( R = [v^2 + x^2]^{1/2} \) the expression for \( \rho_m \) can be simplified as follows

\[
\rho_m = R - md \cos \Theta \tag{10}
\]

where \( \cos \Theta = x/R \). Substitution of eqn (10) into eqn (6), with a limited number of sleepers being taken into account, gives the following expression for the vertical component of ground vibration velocity:

\[
v_x(x, y, \omega) = \frac{V(\omega)}{\sqrt{R}} \exp \left[i(\omega/c_R)md - i(\omega/c_R)md \cos \Theta\right] \tag{11}
\]

where we have neglected the term \( md \cos \Theta \) in the denominator.

It is seen from eqn (11) that maximum radiation takes place if all the exponentials in the sum are equal to one, i.e. the expressions in the square brackets of all exponentials are zeros. This is possible if the train speed \( v \) and the Rayleigh wave velocity \( c_R \) satisfy the condition \( \cos \Theta = c_R/v \), which is similar to the conditions for Mach or Cherenkov radiation. Since the observation angle \( \Theta \) should be real (\( \cos \Theta \leq 1 \)), this implies that \( v \) should be larger than \( c_R \). In this case the ground vibrations are generated as cylindrically attenuated surface waves (factor \( \sqrt{R} \) in the denominator) symmetrically propagating at angles \( \Theta \) with respect to the track, and with amplitudes larger than for 'sub-Rayleigh trains'.

All principal features of the above mentioned remain valid for tracks with an infinite number of sleepers. As was shown in earlier work,\(^5,6\) dissipation of Rayleigh waves in the ground and their geometrical attenuation (factors \( \rho_m^{1/2} \) in eqns (6) and (9)) mean that only about 200 sleepers need to be considered. However, since in this case we deal with the near-field of radiating track, the analytical description is very bulky (like in the Fresnel zone of classical flat radiators), and it is preferable to use direct numerical calculations of formula (6) with the exact expression for the distances \( \rho_m \).

The increase in amplitudes of vibrations for \( v > c_R \) can be explained by two features. The first is the obvious fact that the fields radiated by different sleepers are combined in phase. Therefore, an increase by the number of effectively radiating sleepers of the track, i.e. about 200 times, can be expected compared with the average vibration level for conventional trains. The second feature is the dependence of functions \( P(\omega) \) or \( V(\omega) \)
Fig. 1. Function \((\pi/2)^{1/2} (2\pi F)^{1/2} P(v, F)/(T/d_\pi x_0)\); \(\beta = 1.28 \text{ l/m}\). (a) Mesh: \(\Delta v = 10 \text{ m/s}, \Delta F = 1 \text{ Hz}\); (b) \(v_{\text{max}} = 250 \text{ m/s}, F_{\text{max}} = 50 \text{ Hz}\).
on train speed \(v\) (eqns (4) and (7)). Figure 1 shows the three-dimensional and contour graphs of the function \((\pi/2)^{1/2}(2\pi F)^{3/2} P(v,F)/(T_0/\pi x_0)\), where \(F = \omega/2\pi\). One can see an average increase of about 10 times for \(v = 138.8\) m/s (500 km/h) compared with \(v = 13.88\) m/s (50 km/h). Thus, a total increase of ground vibration amplitudes by 1000–2000 times (60–66 dB) can be expected for the case of trans-Rayleigh trains.

It is interesting to note that according to eqn (11), the amplitudes of the vibration field radiated by parts of the track at angles \(\Theta = \arccos (c_R/v)\) depend neither on the periodicity of sleepers \(d\) nor on their number \(2Q + 1\). They are determined only by the track distance considered. In fact, since the summation in eqn (11) gives \(2Q + 1\) in this case and \(V(\omega)\) is proportional to \(d\) (according to eqns (4) and (7)), the value of \(v_z(x,y,\omega)\) is proportional to the distance \(S = (2Q + 1)d\). This dependence remains valid for the limiting case \(d \to 0\), for constant track distance \(S\). In this case one can replace the sum in eqn (11) by the integral:

\[
v_z(x, y, \omega) = \lim_{d \to 0} \left\{ \frac{V(\omega)}{\sqrt{R}} \frac{1}{d} \exp \left[ i(\omega/c_R) \right] \right\} \times \int_{-S/2}^{S/2} \exp \left[ i(\omega/v)\xi - i \cos \Theta (\omega/c_R)\xi \right] d\xi
\]

where discrete distance \(md\) has been replaced by \(\xi\) and sleeper spacing \(d\) by differential \(d\xi\). Again, since \(V(\omega) \sim d\), we obtain the result that for \(\cos \Theta = v/c_R\) the value of \(v_z(x,y,\omega)\) is proportional to \(S\). This means that radiation of ground vibrations by trans-Rayleigh trains can also take place on tracks without sleepers. Note, however, that for conventional low-speed trains \((v \ll c_R)\), the exponential function inside the integral in eqn (12) oscillates quickly and the value of the integral is close to zero, indicating that ground vibrations in the form of waves are not generated. This agrees with the well known result of elasticity theory\(^{13}\) that, for loads moving along the free surface of an elastic semispace with speed \(v < c_R\), radiated wave-fields do not exist (only localised quasistatic fields can accompany the moving load). Thus, the presence of sleepers is essential for generating ground vibrations by conventional trains due to the mechanism of quasistatic wheel pressure considered here.

### 4 Numerical Calculations and Discussion

Numerical calculations of ground vibrations generated by superfast trains have been carried out according to eqns (6) or (9) for different values of train speed \(v\), number of sleepers \(2Q + 1\), and for different
Fig. 2. Ground vibration spectra (arbitrary linear units) from a single axle load moving with sub-Rayleigh speeds ($c_R = 125$ m/s). (a) Mesh: $\Delta v = 2.5$ m/s, $\Delta F = 1$ Hz; (b) $v_{max} = 100$ m/s, $F_{max} = 50$ Hz.
geometrical parameters of both track and train. Summation over \( m \) in eqns (6) and (9) was carried out from \( m = -Q \) to \( m = Q \). The chosen value of \( Q \) in eqn (9) \((Q = 150)\) was such that the corresponding length of track \((2Q + 1)d\) was greater than the total train length \( NL \) and the attenuation distance of Rayleigh waves at the frequency band considered. The Poisson's ratio of soil was set at 0.25, and the mass density of soil \( \rho_0 \) was set at 2000 kg/m\(^3\).

Figure 2 shows the spectra of ground vibration velocity (in arbitrary linear units) generated by a single axle load moving along the track with sub-Rayleigh speeds, from 2.5 m/s to 100 m/s. The results are shown on three-dimensional (a) and contour (b) plots for the frequency band 0–50 Hz. Units of calculation were \( \Delta v = 2.5 \) m/s and \( \Delta F = 1 \) Hz. The Rayleigh wave velocity \( c_R \) was equal to 125 m/s. Other parameters are: \( \beta = 1.28 \) m\(^{-1}\), \( y_0 = 30 \) m.

One can see that with increase of train speed the ground vibration level grows, especially in the low-frequency area. For relatively low train speeds, the peaks corresponding to the train passage frequencies are clearly seen, indicating linear dependence of the corresponding frequencies on \( v \) (see also the dashed line in Fig. 2(b)). Second harmonics of the main passage frequencies are visible in Fig. 2(a). Note that for train speeds above 12.5 m/s the phenomenon of broadening and splitting of peaks for the main passage frequencies takes place (see also the earlier calculations\(^5\)\(^,\)\(^6\)). This phenomenon can be explained by the phase shifts between waves radiated from different sleepers. These shifts are obviously more pronounced for high speeds and high frequencies when the contribu-

**Fig. 3.** The same as in Fig. 2, but for ground with higher Rayleigh-wave velocity \((c_R = 1250 \text{ m/s})\).
tion of the second term in the sum of eqn (6) is greater. To confirm this explanation, another calculation has been carried out (Fig. 3), with the value of Rayleigh wave velocity ten times greater: \( c_R = 1250 \, \text{m/s} \). Such a value of \( c_R \) is typical for solid clay soils. As expected, in this case the splitting phenomenon disappears.

In Fig. 4, the spatial distributions of ground vibration fields taken at the spectral component \( F = 31.4 \, \text{Hz} \) (vibration velocity or vertical surface displacement) are shown for sub-Rayleigh (a) and trans-Rayleigh (b) speeds, and a single axle load. The fields are displayed in terms of arbitrary linear units. The area of the ground surface considered is \( 48 \, \text{m} \times 48 \, \text{m} \). To demonstrate the formation of wave-fields for both speeds, a small part of the track with just 10 sleepers located in the centre of the area has been considered. The Rayleigh wave velocity \( c_R \) was set as \( 125 \, \text{m/s} \), other parameters being the same as in Fig. 3.

Figure 4 shows clearly that at low speeds (a) the waves are radiated in almost all directions, whereas at trans-Rayleigh speeds (b) the generated

![Fig. 4](image)

**Fig. 4.** Space distribution of the ground vibration field (arbitrary linear units) generated by a single axle load passing over 10 sleepers \((c_R = 125 \, \text{m/s}, F = 31.4 \, \text{Hz}, \text{mesh: } \Delta x = \Delta y = 1 \, \text{m})\). (a) \( v = 13.88 \, \text{m/s} \), (b) \( v = 138.8 \, \text{m/s} \).
wave-field is concentrated mainly in the direction of a train movement, occupying the sector roughly determined by the angles $\Theta = \arccos\left(\frac{c_R}{v}\right)$ with respect to the track. The amplitudes of generated waves are approximately 1000 times larger in (b) than in (a) as can be seen from the vertical scales on the figures.

**Fig. 5.** The same as in Fig. 4(b), but with 100 sleepers being taken into account ($c_R = 125$ m/s, $v = 138.8$ m/s, $F = 31.4$ Hz). (a) Mesh: $\Delta x = \Delta y = 1$ m; (b) $x_{\text{max}} = y_{\text{max}} = 12$ m.
Fig. 6. Ground vibration spectra (arbitrary linear units) generated by a single axle load in a wide range of train speeds. (a) Mesh: $\Delta v = 10$ m/s, $\Delta F = 1$ Hz, other parameters are the same as in Fig. 2; (b) $v_{\text{max}} = 250$ m/s, $F_{\text{max}} = 50$ Hz.
Figure 7. Ground vibration spectra (dB re 10^-9 m/s) generated by a train of five carriages moving with sub-Rayleigh and trans-Rayleigh speeds: \( c_R = 125 \text{ m/s} \), \( L = 8.3 \text{ m} \), \( M = 4.88 \text{ m} \), \( a = 2.2 \text{ m} \), \( T = 100 \text{ kN} \), \( \beta = 1.28 \text{ m}^{-1} \), \( T_{cr} = 108 \text{ kN} \), \( y_0 = 30 \text{ m} \).

Figure 5 shows the spatial distribution of the ground vibration field generated by an axle load moving with trans-Rayleigh speed along a straight track. Here 100 sleepers have been taken into account instead of 10, but otherwise all parameters are the same as for Fig. 4.

It is seen that the effect of a larger number of radiated sleepers is to produce a very high directivity of ground vibration radiation. The vibration field consists of almost perfect plain waves propagating at the angles \( \Theta = \arccos(c_R/v) \) with respect to the track. For the values of \( v \) and \( c_R \) considered, \( \cos \Theta = 0.90 \). This is in excellent agreement with the directions of wave propagation (perpendiculars to the wave fronts) determined from Fig. 5(b).

The ground vibration spectra generated by a single axle load moving with speeds ranging from 10 m/s to 250 m/s, are shown in Fig. 6. These calculations extend the results of Fig. 2 to higher values of train speed \( v \). Note that the details of the spectra for relatively low train speeds shown in Fig. 2(a) are almost invisible in Fig. 6 because of the huge increase of vibration level in the trans-Rayleigh range \( (v \geq c_R) \).

Figure 7 illustrates the ground vibration spectra (in decibels, with regard to the reference level 10^-9 m/s) generated by a train consisting of \( N = 5 \) equal carriages for both sub-Rayleigh and trans-Rayleigh train speeds. The Rayleigh wave velocity in the ground is \( c_R = 125 \text{ m/s} \), and the soil attenuation coefficient has been set to \( \gamma = 0.00478 \). The geometrical parameters of the train (see figure caption) were chosen to be the same as in earlier papers.5,6

One can see that the averaged ground vibration level from a train moving with trans-Rayleigh speed 138.8 m/s (500 km/h) is approximately
70 dB higher than from a train travelling at speed 13.8 m/s (50 km/h). This tremendous increase in ground vibration level agrees well with general analytical estimates given in the previous sections and with results obtained for a single axle load.

5 CONCLUSIONS

Superfast railway trains moving with speeds approaching or exceeding the Rayleigh wave velocity in the ground can cause very large increases in ground vibration level relative to conventional trains. Fortunately, soils with very low Rayleigh wave velocity (around 100 m/s) are uncommon, the most typical range of $c_R$ values being 250–500 m/s. Nevertheless, the designers and builders of tracks for superfast trains should be aware of the potential risk of excessive ground vibrations. One has either to avoid areas such as soft sandy soils with low Rayleigh wave velocity, or to undertake special mitigation measures to protect the built environment from the expected severe ground vibrations.

Measures to reduce ground vibrations from trans-Rayleigh trains need to take into account the peculiarities of ground vibration fields generated by such trains. One such measure might be based on smallness of the radiation angle $\Theta$ for trans-Rayleigh trains in most of the situations. Therefore, if the track is placed inside an open waveguide for surface waves, then most of the radiated energy will be trapped and dissipated in the waveguide without damaging the area outside. Specially modified embankments or trenches can be suggested as possible waveguides, where the total reflection of surface waves incident at sliding angles on the boundary of their top or bottom flat areas would provide the waveguide effect. Waveguides of this kind are similar to topographic waveguides that have been studied intensively for ultrasonic applications over the last 20 years. There is little doubt that similar ideas could be realised for embankments and trenches carrying tracks for superfast trains. A detailed investigation of these problems will be given in a separate paper.

REFERENCES