

## OP-AMP THEORY & APPLICATIONS

### Introduction

Operational amplifiers (op-amps for short) are incredibly useful devices that can be used to construct a multitude of electronic circuits. They are particularly attractive in both amplifier design and instruction, because more often than not, they can be treated as ideal amplifiers, in that they provide a prescribed gain between input and output ports, independent of the circuitry loading those ports. While no amplifier is in fact ideal, the clarity and insight afforded by the assumption of ideal behavior makes the op-amp an attractive first step in designing any amplifier, even those that do not in fact exploit op-amps (e.g., other transistor amplifiers, which we will consider in later labs).

Broadly speaking, op-amps can be used two ways: 1) in the so-called *open-loop* mode, which is useful for comparators and triggers, and 2) with *feedback*, which is how nearly all amplifiers, filters and oscillators using op-amps are designed.

Theory

(Ideal) Op-Amp Basics

Conceptually, an op-amp is nothing more than a voltage-controlled voltage source (VCVS for short) with infinite gain, as shown in shown in Fig. 1. Normally, the op-amp is represented schematically as a triangle with two input terminals and one output terminal; the internal VCVS is implied. Note that in the VCVS representation, the input port is shown as an open circuit; the implication is that the op-amp input impedance is so high that it does not load the circuitry driving the op-amp. If the op-amp input impedance were comparable to the output impedance of the driver circuitry, there would be a significant voltage divider at the input port, not only reducing the op-amp output signal, but causing the op-amp output to vary directly with the preceding stage output impedance; this is undesirable, as the op-amp performance should depend entirely on its own characteristics, not those of external, and hence variable, components. Likewise, the ideal op-amp has no series output impedance, meaning that the op-amp can drive any load without voltage-divider attenuation. Further notice that since the op-amp output port consists of a voltage source, there is no limit on the op-amp output current.

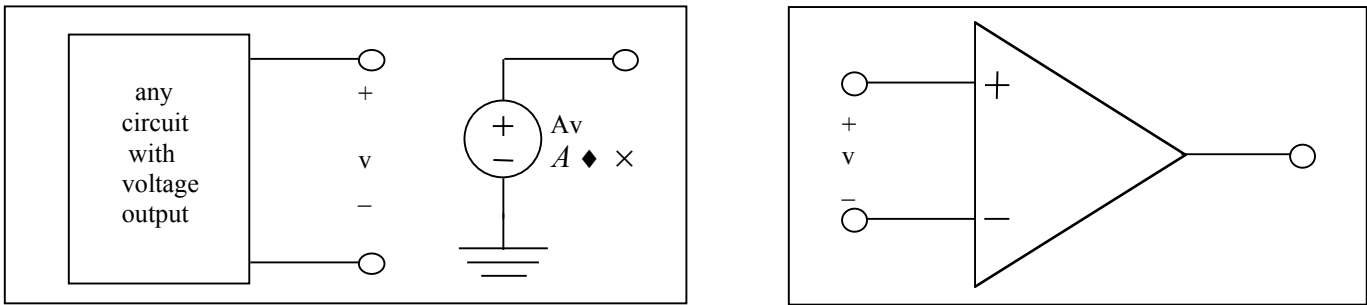
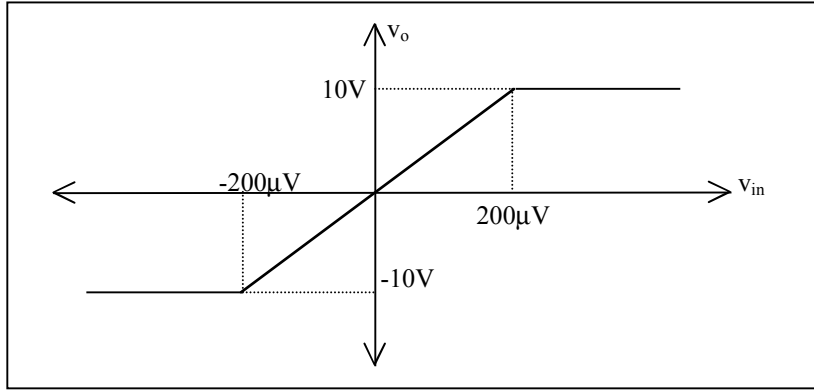


Fig. 1 (a) VCVS conceptual representation of op-amp, (b) schematic representation.

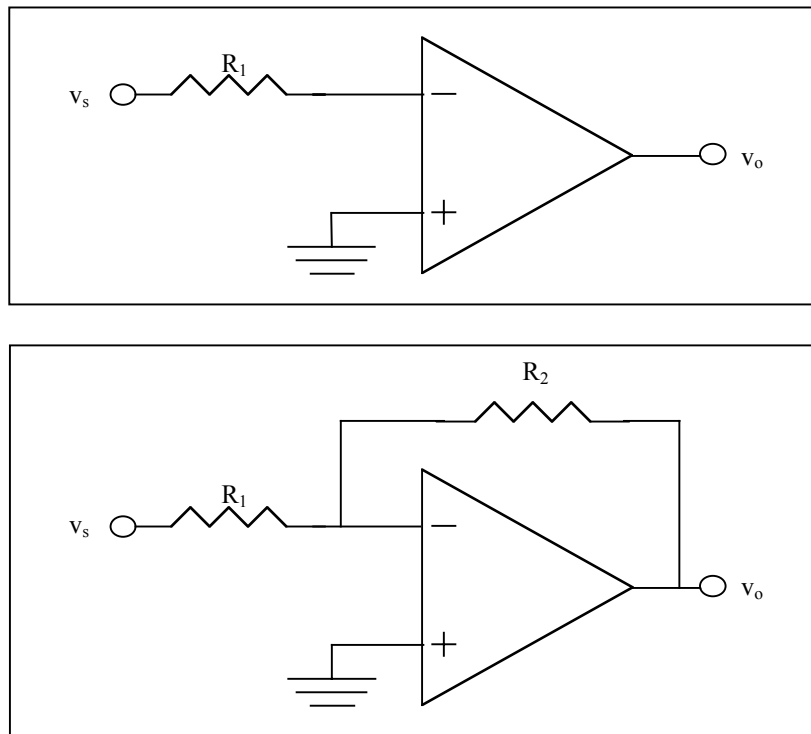
At this point, you may be wondering of what use is a voltage source with infinite gain. Unless the input voltage is exactly zero, won't the output voltage fry the electronics at the output port? This dilemma is resolved by the observation that the op-amp is a VCVS, an *active element*, necessitating an energy source to supply the gain. This source is the power supply, which is limited to a few volts (for op-amps in the lab, this may be +/- 12 V, on integrated circuits, perhaps as low as +/- 1.5 V), constraining the output voltage to reside within the power supply boundaries. For everything to make sense then, the "infinite" gain is only large for very small inputs signals, and for input voltages greater than this threshold, the output simply clips at one of the supply rails, determined by whether the input signal is positive or negative. This behavior is shown in Fig. 2.



**Fig. 2** Behavior of op-amp output voltage, assuming VCVS gain = 94 dB, power supply =  $\pm 10\text{V}$ .

**Feedback**

For the situation of Fig. 2, it is clear that the VCVS acts linearly only for inputs less than  $\pm 200\mu\text{V}$ ! This is so small compared to the supply as to seem insignificant. It would appear that to be practical, one might just as well ignore the linear region, modelling the VCVS as  $10 \cdot \text{sgn}(v_{in})$ . In fact, with the exception of comparator/trigger-type functions, this is the *only* region in which the op-amp is used! Utilizing feedback, in which a portion of the op-amp output is fed back to the inverting terminal, the op-amp will produce an output signal that varies proportional to the input signal by *forcing* the two input terminals to be at nearly the same potential. Consider the situation depicted in Fig. 3:



**Fig. 3** (a) Op-amp without feedback, (b) with feedback.

Without feedback, the op-amp output lies at the supply rail. However, when resistor  $R_2$  is connected, the following Kirchhoff equations can be written:  $\frac{v_i + v}{R_1} = -\frac{v + v_o}{R_2}$ ,  $Av = v_o$ , resulting in

$$\frac{v_o}{v_s} = -\frac{R_2/R_1}{1 + \frac{R_2/R_1}{A}} \cup -\frac{R_2}{R_1}$$

Suppose now that  $v_s$  is a sine wave with amplitude 100 mV; while the amplitude of the source voltage is *much* more than the permissible 200  $\mu$ V range, the feedback path provided by the  $R_1$ - $R_2$  voltage divider (recognize that shorting out  $v_i$  and treating  $v_o$  as the source, the signal fed back to the inverting terminal is  $v_o \frac{R_1}{R_1 + R_2}$ ) causes the op-amp to realize a net input differential very close to zero with an output amplitude of 1 V, which is much less than the power supply.

### Op-Amp Metrics

So far, a very simple model of the op-amp has been explored, and in general this is quite suitable for many designs, and certainly adequate as a first-pass at almost any design. However, real-life designs often have stringent specifications that force one to consider more complicated, non-ideal op-amps. Because of this, it is worth the exploring the sources of some common non-idealities to see what can be done to mitigate their effects.

### Slew Rate

The term slew rate refers to how fast the output can swing without becoming distorted. Consider the circuit discussed in Fig. 3(b), again with a sinusoidal input. Recall that the slope of a sinusoid is directly proportional to its frequency and amplitude; this means that as one increases the amplitude and/or frequency of the input sinusoid and/or the closed-loop gain of the op-amp, the output voltage must swing at a faster and faster rate.

It turns out that the input stage of an op-amp is a transconductor, which ideally converts the input signal voltage into a proportional signal current. However, in reality, as the input voltage swing gets too large, the signal current available from the transconductor approaches an asymptotic limit, which is a non-linear phenomenon. It follows that even if the signal current is run through a *perfect* current-to-voltage converter to produce the final output voltage, the op-amp output voltage is necessarily *not perfectly proportional* to the input voltage, and will appear distorted.

Furthermore, the output of any op-amp is capacitive to some degree, which complicates matters still more.

Recalling that the i-v relation of a capacitor is  $\frac{dV}{dt} = \frac{I}{C}$ , it is clear that the rate at which the output

voltage can swing is diminished for large output capacitance and small charging current (this charging current is limited by how much DC current the input stage is able to provide the output stage, which ties directly to the limiting signal current phenomenon discussed earlier).

While the input voltage may increase very rapidly, the output climbs a bit slower; once the input sinusoid has reached its peak and begins to decrease, the output naturally wants to reach its peak, though the input is directing it to change direction and decrease. Simply put, slew rate is electrical momentum – at low speeds, it is easy to change the direction of a moving object, but the faster that object moves, the longer the response time for the object to pursue the direction dictated by the driving force.

## Bandwidth

Like slew rate, the finite bandwidth of an op-amp limits the performance at high frequencies. However, whereas slew rate non-linearly *distorts* the shape of a given output frequency (i.e., alters its shape from that of the input), the bandwidth of the op-amp causes the output amplitude to decrease with increasing frequency, as well as incur some phase shift relative to the input, which is a linear effect. The culprit is typically compensation capacitance internal to the op-amp, provided to guarantee a one-pole response, and as we will see later, guarantee a stable response when feedback is applied. To account for this effect, one may replace the gain “A” with a frequency-dependent gain,  $A/(1 + s/p)$ . One final note is that the finite bandwidth can cause *linear distortion* if the input waveform consists of more than one sinusoid, each experiencing a different phase shift and amplitude reduction due to the one-pole response. While the response is still linear, the output waveform may look completely different from the input.

## Offset Voltage

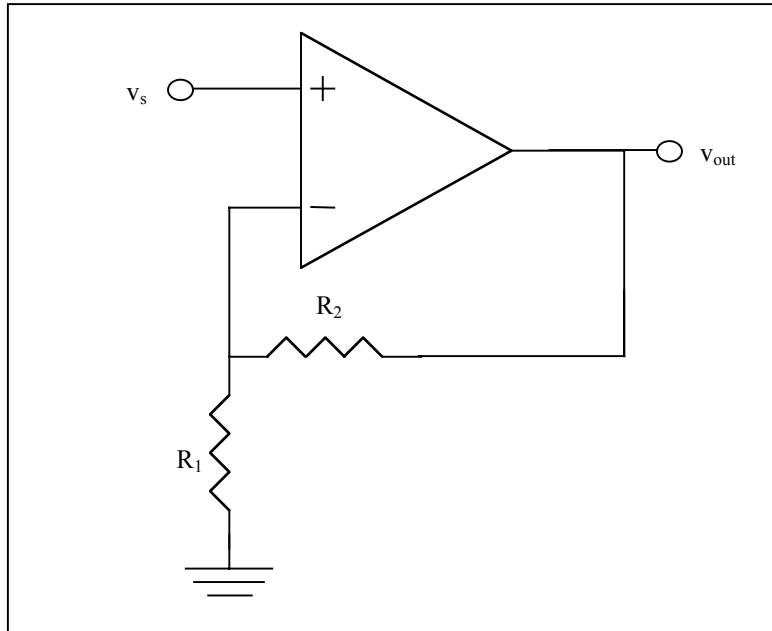
Assuming the op-amp is totally balanced inside, then when both input terminals are at the same potential, the output voltage is precisely zero. However, in reality, the circuitry looking into both terminals is not precisely matched (due both to asymmetry in the input stage of the op-amp as well as processing imperfections in manufacturing the op-amp), and this results in an innate imbalance leading to an offset voltage. The offset voltage of an op-amp is defined as the input voltage differential required to bring the output voltage to zero. Note that as long as this input voltage lies within the range necessary to cause linear VCVS gain, one can relate the input offset voltage to the output voltage resulting from zero input by the gain “A” of the op-amp.

**Reference Reading**

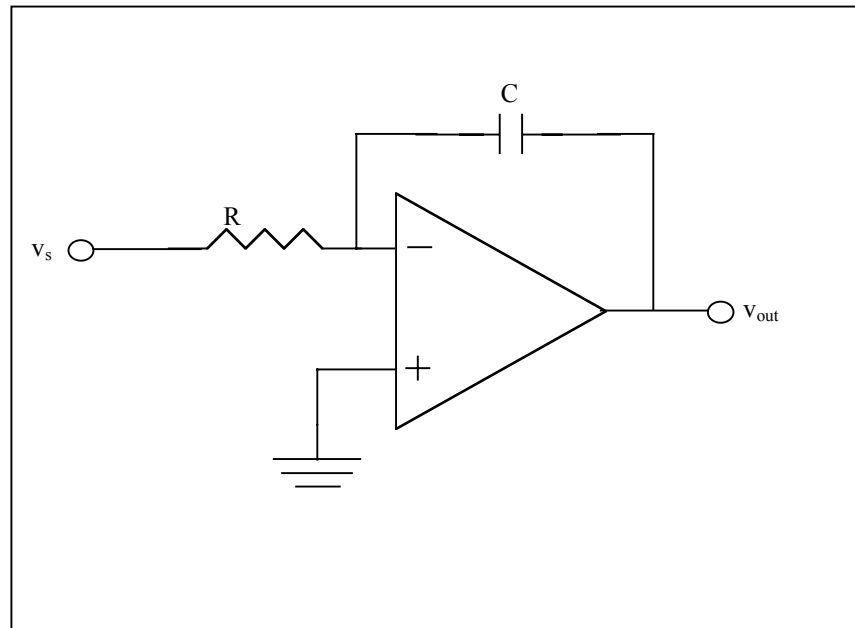
- 1) Roland E. Thomas & Albert J. Rosa. *The Analysis and Design of Linear Circuits*, chapter 4. Prentice-Hall Inc., Englewood Cliffs, New Jersey, 1994.
- 2) Donald A. Neamen. *Electronic Circuit Analysis and Design*, chapters 9 & 13. Irwin, Chicago, 1996.
- 3) David Johns & Ken Martin. *Analog integrated Circuit Design*, chapters 5 & 6. John Wiley & Sons, Inc., New York, 1997.
- 4) Paul R. Gray & Robert G. Meyer. *Analysis and Design of ANALOG INTEGRATED CIRCUITS*, chapter 6. John Wiley & Sons, Inc., New York, 1993.

**Prelab Exercises**

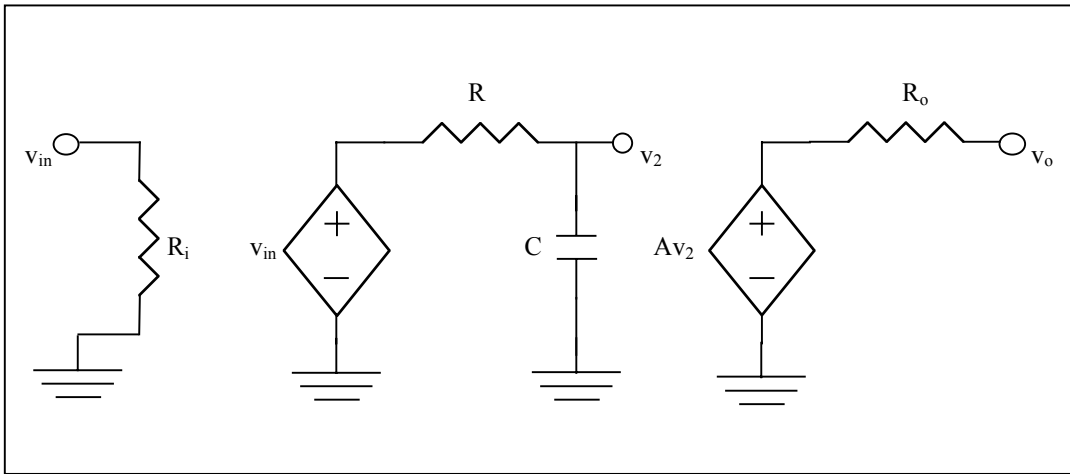
- 1) Compute the gain of the following amplifier, assuming an ideal op-amp. What are the key differences in the gain of this op-amp compared to the inverting amplifier considered earlier? What range of gains can one achieve with this configuration?



- 2) Consider the op-amp circuit below. Derive the time-domain input-output relationship, and write it in the form  $v_o = f(v_s)$ . Considering your response, what does this circuit do? What is the purpose of  $R_2$ ? Derive the frequency domain transfer function. Plot the magnitude of the transfer function on log-log axes, and the phase on linear-log scale axes. How would these sketches differ if  $R_2$  were absent?



- 3) Consider the above circuit, but swapping the locations of resistor and capacitor. Repeat Prelab exercise 2.
- 4) Consider the following model of an op-amp. Analytically, determine the DC gain, bandwidth and unity-gain frequency of the op-amp. (The unity-gain frequency is that frequency at which the transfer function magnitude decreases to 0 dB). For a single-pole model such as this, what is the transfer function,  $v_o/v_{in}$ ? Identify the DC gain,  $G$ , and the 3-dB bandwidth,  $BW$ , and simplify these results for very large input resistance and very small output resistance. What is the unity-gain frequency predicted by this single-pole model? Assuming that the ratio of unity-gain frequency to 3-dB bandwidth is large, what is a reasonable approximation to the unity-gain frequency (express your final result in terms of  $G$  and  $BW$ )?



- 5) Consider an op-amp hooked up in the classical inverting configuration; assuming the op-amp follows the same single-pole model given in the previous question, determine the transfer function,  $v_o/v_{in}$ . Identify the DC gain,  $G_2$ , and the 3-dB bandwidth,  $BW_2$ , and simplify these results for very large input resistance and very small output resistance. What is the ratio of the DC gain found in this problem to that found in the preceding question? What is the ratio of the 3-dB bandwidths? Assuming the unity-gain frequency is much larger than the 3-dB bandwidth, what is the unity-gain frequency? How does this result compare to that found in question 4? This is a very important result that holds for any single-pole system with scalar feedback.
- 6) This question deals (very, very crudely) with slew rate in op-amps. Consider once more the op-amp model in question 4, hooked up in an inverting configuration and two strings of diodes between the op-amp output and ground, each in opposite polarity. Simulate this model in SPICE (do a time-domain analysis, not an AC analysis!) assuming  $R_i=10M\Omega$ ,  $R_o=100\Omega$ ,  $R=1.6M\Omega$ ,  $C=10pF$ ,  $A=5,000$ , a gain of  $-10$  (i.e.,  $R_2/R_1 = 10$ ). To emulate the slewing condition, first implement the output dependent source in SPICE as:

Eout in+ in- out+ out- 5000 max=5 min=-5

and second, each diode string should have  $5V/700mV \sim 7$  diodes. The max/min statements cause the op-amp to appear as if it were powered by  $\pm 5V$  supplies. When the input is negative, the output goes positive, and as the output approaches the positive supply, one of the diode strings turns on, causing the output to limit. When the input goes positive, the other diode string kicks in as the output approaches the negative supply.

Simulate this circuit, varying the frequency and amplitude of the source voltage, and determine the rising and falling slew rates as the slope of the output waveform in the neighborhood of adjacent zero crossings.

### Lab Exercises

- 1) Build the circuit in prelab exercise 1, and verify its behavior.
- 2) Build the circuit in prelab exercise 2, and verify its behavior.
- 3) Build the circuit in prelab exercise 3, and verify its behavior.
- 4) Determine the slew rate of both a 741 and LM301 op-amp. Connect the op-amps in inverting configurations (gain of  $-10$ ), and apply a sinusoidal input. For frequencies of 100Hz, 1kHz, 10kHz and 100kHz, vary the amplitude and monitor the output voltage on the oscilloscope. Determine the slew rate (this will be pretty rough, but again, we're just trying to get the concept) from the slope at the zero crossings as the waveform starts to distort. Can the spectrum analyzer be used to determine the frequency above which slewing is a problem for a fixed closed-loop gain?
- 5) Determine the unity-gain frequency of the 741 op-amp. Is there a better way than trying to measure it directly? (Hint: consider the approximation you determined in Prelab exercise 4). Compare your results from a spectrum analyzer and an oscilloscope measurement. Which is likely more accurate? How might you increase the accuracy of your reading on the less accurate instrument?