**Problem #1:**

![Circuit Diagram]

\[
I_1 = \frac{V_1}{R_i} - g_2 V_2 = \frac{V_1}{R_i} + \frac{g_1 g_2 R_i V_1}{1 + sR_0 C}
\]

\[
Y_{in}(s) = \frac{I_1}{V_1} = \frac{1}{R_i} + \frac{g_1 g_2 R_0}{1 + sR_0 C}
\]

**Ans. #1**

**Problem #2:**

\[
Y_{in}(s) = \frac{1}{R_i} + \frac{1}{R_2 + sL} = \frac{1}{R_i} + \frac{1}{1 + sL/R_2}
\]

\[
P_1 = R_i, \quad \frac{L}{R_2} = R_0 C
\]

\[
P_2 = \frac{1}{g_1 g_2 R_0}, \quad L = R_2 R_0 C = \frac{C}{g_1 g_2}
\]

**Ans. #2**

**Problem #3:**

(a) For \( R_i, R_0 \to \infty \), \[ Y_{in}(s) = \frac{g_1 g_2}{sC} \]

Since the admittance of an inductor is \( Y(s) = \frac{1}{sL} \), the gyrator ideally yields an inductance of value \( C/g_1 g_2 \).
(b). With \( R_1, R_0 \to \infty \), \( R_1 = \infty \); \( R_2 = 0 \), meaning that the passive model is a simple inductance. \( \text{Ans.} \#56 \)

(c). For a series RL circuit, \( Z_{in} = R + j \omega L = R + j \omega X \), where \( X > 0 \), assuming an RL circuit:

\[
\begin{align*}
Y_{in}(j\omega) &= \frac{1}{R_i} + \frac{j\omega g_2 R_0}{1 + j\omega R_0 C} = \frac{1 + j\omega R_0 C + j\omega g_2 R_0 R_0}{R_i (1 + j\omega R_0 C)} \\
Z_{in}(j\omega) &= \frac{\frac{R_i}{(1 + j\omega R_0 C)} (1 + j\omega R_0 C)}{(1 + j\omega g_2 R_0 R_0) + j\omega R_0 C} \\
\Rightarrow Z_{in}(j\omega) &= \frac{\frac{R_i}{1 + j\omega R_0 C} (1 + j\omega R_0 C)}{1 + j\omega R_0 C} = \frac{\frac{R_i}{1 + j\omega R_0 C} (1 + j\omega R_0 C)}{1 + (\frac{\omega R_0 C}{R_i})^2} = \frac{\frac{R_i}{1 + j\omega R_0 C} (1 + j\omega R_0 C)}{1 + (\omega R_0 C)^2} = R_{in}(w) + jX_{in}(w)
\end{align*}
\]

\[
\begin{align*}
\Omega(w) &= \frac{\omega R_0 C (1 - \frac{1}{K})}{1 + (\frac{\omega R_0 C}{K})^2} = \frac{\omega R_0 C (K - 1)}{K + (\omega R_0 C)^2} = \frac{\omega g_1 g_2 R_0 R_0}{1 + g_1 g_2 R_0 R_0 + (\omega Y)^2}
\end{align*}
\]

\[
\begin{align*}
\Omega(w) &= \frac{\omega R_0 C (1 - \frac{1}{K})}{1 + (\frac{\omega R_0 C}{K})^2} \quad \Rightarrow \quad \Omega(0) = \Omega(\infty) = 0
\end{align*}
\]

\[
\begin{align*}
\lim_{w \to 0} \Omega(w) &= \omega R_0 C (K - 1) = \omega g_1 g_2 R_0 R_0^2 C \\
\lim_{w \to \infty} \Omega(w) &= \frac{K - 1}{\omega Y} = \frac{g_1 g_2 R_0}{\omega C} \quad \text{Ans.} \#53C
\end{align*}
\]
\( L = \frac{C}{g_1 g_2} \Rightarrow g_1 g_2 = \frac{C}{L} = (1)(10^{-8}) \text{ mho}^2 \)

\( R_i = R_o = 10 \text{ k}\Omega \Rightarrow K = 1 + g_1 g_2 R_i R_o = (100)(10^8) \)

\( T = R_o C = 100 \text{ nsec} \)

\[ Q(w) = \frac{wY(K-1)}{K + (wY)^2} \]

\[ Q(x) = \frac{(K-1)x}{K + x^2} \]

\[ \frac{dQ(x)}{dx} = \frac{(K+x^2)(K-1) - (K-1)x(2x)}{(K+x^2)^2} = 0 \]

\[ (K+x^2)(K-1) = 2x^2 (K < 1) \]

\[ x^2 = K \]

\[ x = wY = \sqrt{K} \Rightarrow w = 2\pi(503.3 \text{ MHz}) \]

\[ Q_{\text{max}} = \frac{\sqrt{K}(K-1)}{2K} = \frac{K-1}{2\sqrt{K}} = 158.1 \] (See Plot, Page 8)

**Problem #4:**

\[ \frac{V_0}{R_i} + S g_2 (V_0 - V_x) = 0 \Rightarrow V_x = \frac{(1 + S R_i C_2)V_0}{S R_i C_2} \]
\[ S(C_1)(V_x - V_0) + S(C_2)(V_x - V_0) + \frac{V_x - V_0}{R_2} = 0 \]

\[ V_x \left\{ 1 + SR_2C_1 + SR_2C_2 \right\} - V_0 \left\{ 1 + SR_2C_2 \right\} = SR_2C_1 V_x \]

\[ \frac{1 + SR_2C_2}{SR_2C_2} \left\{ (1 + SR_2C_1 + SR_2C_2) - 1 - SR_2C_2 \right\} V_0 = SR_2C_1 V_x \]

\[(1 + SR_2C_2)(1 + SR_2C_1 + SR_2C_2) - SR_2C_1 - S^2 R_1 R_2 C_2) V_0 = SR_2C_1 V_x \]

\[ H(s) = \frac{V_0}{V} = \frac{S^2 R_1 R_2 C_1 C_2}{1 + SR_2C_1 + SR_2C_2 + S^2 R_1 R_2 C_2} \]

\[ = \frac{S^2 R_1 R_2 C_1 C_2}{1 + SR_2(C_1 + C_2) + S^2 R_1 R_2 C_2} \]

\[ \Rightarrow \text{Highpass Filter} \]

\[ (b). \quad \omega_N^2 = \frac{1}{R_1 R_2 C_1 C_2} \Rightarrow \omega_N = \sqrt{\frac{1}{R_1 R_2 C_1 C_2}} \]

\[ \frac{2\rho}{\omega_N} = R_2(C_1 + C_2) \Rightarrow \rho = \frac{R_2(C_1 + C_2)}{2} \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \]

\[ \rho = \frac{1}{2 \sqrt{R_2}} \left( \frac{C_1 + C_2}{C_1 C_2} \right) \]

\[ (c). \quad H(j\omega) = -\frac{(\omega/\omega_N)^2}{1 - (\omega/\omega_N)^2 + j2\rho(\omega/\omega_N)} \]
\[ H(j\omega_n) = -\frac{1}{j2p} \]

With \( p = \frac{1}{\sqrt{2}} \) \( \Rightarrow \) \( H(j\omega_n) = -\frac{1}{j\frac{1}{\sqrt{2}}} \) \( \Rightarrow \) \( |H(j\omega)| = \frac{1}{1/\sqrt{2}} \)

Since the infinite frequency gain is one and since the gain at frequency \( \omega_n \) is down from this infinite frequency gain by \( \frac{1}{\sqrt{2}} \) (3-dB), \( \omega_n \) is the low 3-dB cutoff frequency when \( p = 1/\sqrt{2} \)

\[ \text{Ans.: #40} \]

(d). Let \( x = \omega/\omega_n = \text{normalized frequency} \)

\[ H(jx) = -\frac{x^2}{1-x^2 + j2px} \] \( \Rightarrow \) \[ |H(jx)| = \frac{x^2}{(1-x^2)^2 + (2px)^2} \]

\[ \Phi_H(x) = 180^\circ - \tan^{-1}\left(\frac{2px}{1-x^2}\right) \]

See plots, pages 11-12. Note that for small \( p \), the magnitude response displays pronounced peaking in the circuit response. Small \( p \) also provides a lower low cutoff frequency. For large \( p \), no peaking is evidenced, but the low cutoff frequency is larger. As a general design guideline, \( p = 1/\sqrt{2} \) is desirable in that it delivers the flattest possible magnitude response within the constraint of no peaking. The effect of \( p \) on the phase response is less dramatic, but observe that for small \( p \), the sensitivity of phase angle is greater than it is for large \( p \), particularly in the neighborhood of unity normalized frequency, i.e., around \( \omega = \omega_n \).
Problem #5:

(a) \( \frac{1}{j \omega C} \frac{1}{j \omega L} = \frac{\frac{R}{1+j \omega RC}}{1+j \omega RC} \cdot \frac{\frac{1}{R+j \omega L+(j \omega)^2 RC}}{R+j \omega L+(j \omega)^2 RC} \)

\[ = \frac{\frac{R}{1+j \omega L+(j \omega)^2 RC}}{1+j \omega L+(j \omega)^2 RC} \cdot \frac{R}{1+j \omega L(w L C - \frac{1}{w})} \]

Let: \( \omega_0 = \frac{1}{\sqrt{LC}} \); \( \omega_0 = \frac{R}{w_0 L} \)

\[ \Rightarrow \frac{1}{j \omega C} \frac{1}{j \omega L} = \frac{R}{1+j \omega_0(\frac{w}{w_0} - \frac{w_0}{w})} \]

\[ \Rightarrow H(j \omega) = \frac{V_o}{V_i} = \frac{\frac{V_o}{V_i}}{1+j \omega_0(\frac{w}{w_0} - \frac{w_0}{w})} \]

\( H(j \omega) = 0 \)
\( H(j \omega) = 0 \)
\( H_{\text{max}} = \frac{GmR e^{-j \omega}}{w = \omega_0 = 1/\sqrt{LC}} \)

\( \text{Ans.} \)

(b) \( |H(j \omega)| = |H(j \omega)| = \frac{GmR \omega_{\text{max}}}{\sqrt{2}} = \frac{H_{\text{max}}}{\sqrt{2}} \)

\( \omega_0 \left( \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right) = 1, \text{ Assuming } \omega_H > \omega_0 \)
\( \omega_0 \left( \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega_L} \right) = -1, \text{ Assuming } \omega_L < \omega_0 \)
\[ Q_0 (w_h^2 - w_0^2) = w_0 w_h \]
\[ Q_0 w_h^2 - w_0 w_h - Q_0 w_0^2 = 0 \]
\[ w_h = \frac{w_0 \pm \sqrt{w_0^2 + 4 Q_0^2 w_0^2}}{2 Q_0} = \frac{w_0}{2 Q_0} \left[ 1 \pm \sqrt{1 + 4 Q_0^2} \right] \]

\[ Q_0 (w_L^2 - w_0^2) = -w_0 w_L \]
\[ Q_0 w_L^2 + w_0 w_L - Q_0 w_0^2 = 0 \]
\[ w_L = \frac{-w_0 \pm \sqrt{w_0^2 + 4 Q_0^2 w_0^2}}{2 Q_0} = \frac{w_0}{2 Q_0} \left[ -1 \pm \sqrt{1 + 4 Q_0^2} \right] \]

\[ B = w_h - w_L = \frac{w_0}{Q_0} = \frac{w_0^2 L}{R} = \frac{1}{RC} \quad \text{ANS. } \frac{1}{5 b} \]

(c) \[ Q = \frac{w_0}{B} = \frac{w_0}{w_0^2 L} = \frac{R}{w_0} = R \sqrt{\frac{L}{C}} \]

\[ H_N(j\omega) = \frac{1}{1 + j Q (X - \frac{1}{X})} \]

\[ |H_N(j\omega)| = \frac{1}{\sqrt{1 + Q^2 (X - \frac{1}{X})^2}} \]
\[ \Phi(X) = -\tan^{-1} Q (X - \frac{1}{X}) \]

Let \( H_N = \frac{H(j\omega)}{H_{max}} \) be Normalized Gain
\[ X = \frac{\omega}{w_0} \text{ Normalized Frequency} \]

\[ H_N(j\omega) = \frac{1}{1 + j Q (X - \frac{1}{X})} \]

See plots, pages 15, 16

The larger \( Q \) is, the more peaked is the gain at zero freq, i.e., the gain is sharply defined in the neighborhood of \( w_0 \).