Problem 1

a) The worst case rise time is when only one pmos transistor is on, and it will have to be balanced with 5 nmos transistors in series.

\[ \beta_n = 4.3 \cdot \beta_p \]

\[ \beta_{neff} = \frac{1}{5} \beta_n \cdot \frac{W_n}{L_n}, \quad \beta_{peff} = \frac{W_p}{L_p} = \frac{1}{4.3} \beta_n \cdot \frac{W_p}{L_p} \]

We want \( \beta_{neff} = \beta_{peff} \). Assume \( W_p = 4\lambda \) (We will be sizing the nmos transistors only). Assume \( L_n = L_p = 2\lambda \).

\[ \frac{1}{5} \beta_n \cdot W_n = \frac{1}{4.3} \beta_n \cdot W_p \]

\[ \therefore W_n = 5 \cdot \frac{4\lambda}{4.3} = 4.65\lambda \]

We’ll round up, say \( W_n = 5\lambda \).

b) In the worst case rise time, only the nmos transistor closest to ground is off.
**Problem 1b.**

The worst case fall time is when only one nmos transistor is on, and it will have to be balanced with 8 pmos transistors in series.

\[
\beta_{\text{peff}} = \frac{1}{8} \beta_p \cdot \frac{W_p}{L_p} = \frac{1}{8 \cdot 4.3} \beta_n \cdot \frac{W_p}{L_p}, \quad \beta_{\text{neff}} = \beta_n \cdot \frac{W_n}{L_n}
\]

Again, we want \( \beta_{\text{neff}} = \beta_{\text{peff}} \). Assume \( W_n = 4\lambda \) (We are now sizing the pmos transistors only). Assume \( L_n = L_p = 2\lambda \).

\[
\frac{1}{8 \cdot 4.3} \beta_n \cdot W_p = \beta_n \cdot W_n
\]

\[\therefore W_p = 8 \cdot 4\lambda \cdot 4.3 = 137.6\lambda\]

We’ll again round up, giving us \( W_p = 138\lambda \).

---

**Problem 2**

The worst case fall time is when only one nmos transistor is on, and it will have to be balanced with 8 pmos transistors in series.

\[
\beta_{\text{peff}} = \frac{1}{8} \beta_p \cdot \frac{W_p}{L_p} = \frac{1}{8 \cdot 4.3} \beta_n \cdot \frac{W_p}{L_p}, \quad \beta_{\text{neff}} = \beta_n \cdot \frac{W_n}{L_n}
\]

Again, we want \( \beta_{\text{neff}} = \beta_{\text{peff}} \). Assume \( W_n = 4\lambda \) (We are now sizing the pmos transistors only). Assume \( L_n = L_p = 2\lambda \).

\[
\frac{1}{8 \cdot 4.3} \beta_n \cdot W_p = \beta_n \cdot W_n
\]

\[\therefore W_p = 8 \cdot 4\lambda \cdot 4.3 = 137.6\lambda\]

We’ll again round up, giving us \( W_p = 138\lambda \).
The worst case fall time is when ABCD EFGH = 1000 0000. The only rise time is when ABCD EFGH = 0000 0000.

**Problem 3**

No, the margins are not adequate. If there is 0.4V of noise on the output of the off-chip inverter, an output high can become as low as 1.9V, and an output low can become as high as 0.8V. The on-chip receiving inverter would not interpret either of these correctly. In fact, the circuit will not work correctly even without noise for low values, if the driver low is too high, because the receiver will not see it as a low.
Problem 4

a)
Any path that gives a high output will be a critical path with three transistors in this compound gate. We will use the transition from $ABC = 100 \Rightarrow ABC = 101$ as the worst case rising transition, and the reverse would be a worst case fall transition. The critical path for rise time would be $C, B, \bar{A}$ and the critical fall path would be $A, B, \bar{C}$.

b)

\[ \beta_{neff} = \frac{1}{3} \beta_n \cdot \frac{W_n}{L_n} \]

We want:

\[ \beta_{neff} = \beta_n \cdot \frac{4\lambda}{2\lambda} \]

We will leave $L_n = 2\lambda$.

\[ \therefore W_n = 4\lambda \cdot 3 = 12\lambda. \]

\[ \beta_{peff} = \frac{1}{3} \beta_p \cdot \frac{W_p}{L_p} \]

We want:

\[ \beta_{peff} = \beta_p \cdot \frac{4 \cdot 4\lambda}{2\lambda} \]

We will leave $L_p = 2\lambda$.

\[ \therefore W_p = 4 \cdot 4\lambda \cdot 3 = 48\lambda. \]

c)
We already sized the $\bar{A}$ transistor with $W = 48\lambda$ and $L = 2\lambda$. Since an alternate critical rise path is $ABC = 000$, also with three transistors, we have to size the $A$ transistor the same as the $\bar{A}$ transistor.
As we can see in the figure, there will be 27 diffusion capacitances (blue circles) charging in the worst case.

We will assume here that all the transistors have $1/3R_{chn}$ resistance. Using the lumped model:

$$RC = 3(1/3R_{chn}) \cdot (27C) = 27R_{chn}C.$$
Problem 5

\[ C_g = \epsilon_0 \epsilon \frac{A}{d} \]

\[ = \frac{8.85 \times 10^{-14} F \cdot 100 cm}{m \cdot 3.9 \cdot 2.5(3\lambda) \cdot 2(2\lambda) \cdot (\frac{0.25 \mu}{\lambda})^2 \cdot (\frac{1e - 6m}{\mu})^2 \cdot \frac{1}{57A} \cdot \frac{A}{1e^{-10} m}} \]

\[ = 1.1354e^{-14} F = 11.354 fF \]

Problem 6

\[ P = (2(3\lambda) + 12\lambda) \cdot \frac{25\mu}{\lambda} = 4.5\mu \]

\[ A = (0.1\mu) \left( 12\lambda \cdot \frac{0.25\mu}{\lambda} \right) + (3\lambda)(12\lambda) \cdot \left( \frac{0.25\mu}{\lambda} \right)^2 = 2.55\mu^2 \]

\[ C_d = C_{jbsn} \cdot A + C_{jbswn} \cdot P \]

\[ = 17.27e^{-4} \frac{pF}{\mu^2} \cdot 2.55\mu^2 + 4.17e^{-4} \frac{pF}{\mu} \cdot 4.5\mu = 0.005pF = 5.0 fF \]