Lecture Summary

Lec 14 Hermitian, Unitary and Quadratic Forms

- example of Hermitian matrix (continued)
- eigenvectors of Hermitian matrix associated to distinct eigenvalues are orthogonal
- unitary matrices
- defn and example;
- the eigenvalues of unitary matrix are of unit magnitude
- example
- the eigenvectors associated to distinct eigenvalues are once again, pairwise orthogonal
- unitary matrices preserve length; any matrix that preserves length has to be a unitary matrix;
- example of Hermitian matrix that has an orthonormal set of eigenvectors;
- defn of a normal matrix;
- a matrix is normal iff it has a complete set of orthonormal eigenvectors;
- how does one go about finding a set of orthonormal eigenvectors? procedure outlined;
- quadratic forms: motivation and example;

Lec 13 More on eigenvalues

- example matrices along with their eigenvalues and eigenvectors;
- diagonalization of a matrix;
- defn of a diagonalizable matrix;
- $A = SAS^{-1}$
- application to the Fibonacci sequence
- application of diagonalization to linear, homogeneous, constant coefficient differential equations
- $e^{At}$, $e^{A}$
- Hermitian matrices; defn; the eigenvalues are always real;

Lec 12 Eigenvalues and Eigenvectors

- completing discussion of changes needed in going from linear algebra over the real numbers to over the complex numbers;
• orthogonality in the complex case; Cholesky decomposition

• eigenvalues and eigenvectors: defn of eigenvalue

• example; how to compute eigenvalues

• every $n \times n$ matrix has $n$ eigenvalues (counting multiplicities)

• defn of eigenvectors; how to compute eigenvectors;

• $\text{trace}(A) = \sum \lambda_i$ and $\text{det}(A) = \prod \lambda_i$

• eigenvectors associated to distinct eigenvalues are linearly independent

• handout (attached 2 page) on working with complex numbers;

Lec 11 Determinants

• this lecture was conducted from a handout

• defn of a determinant in terms of $n$ linearity and alternating property; we assume that the formula for the $2 \times 2$ determinant is known; this helps provide example throughout;

• explaining the term ”alternating”

• derived properties of determinants;
  – two linearly dependent rows implies zero determinant
  – interchanging rows changes sign;
  – the elementary row operation that leaves the determinant unchanged
  – a matrix is nonsingular iff det is not equal to zero
  – $\text{det}(AB) = \text{det}(A)\text{det}(B)$ and $A, A^t$ have same determinant (proof omitted);

• formulae for the determinant: in terms of pivots; in terms of permutations; in terms of co-factors;

• applications: finding the inverse of $A$; Cramer’s rule; finding area of parallelogram;

• linear algebra over the complex numbers: most things remain the same; however teh amterial on geometric notions has to be revisited; here the defn of length and inner product etc have to be changed; essentially one repalces the transpose by the Hermitian operator ($\dagger$);

Lec 10 Gram-Schmidt Orthogonalization

• application of LSA to teh digital compression fo speech signals;

• Gram-Schmidt orthogonalization: we approach this projections;

• QR decomposition via the Gram-Schmidt process;
• detour (needed to fully explain $QR$ decomposition): coordinate representation wrt orthonormal basis;

• proof that every set of pairwise orthogonal vectors is a linearly independent set;

• defn of an orthogonal matrix (these matrices are real version of unitary matrices that will be studied later)

• an application to space-time codes;

Lec Least Squares Approximation (continued)

• application of LSA to curve-fitting using polynomials;

Lec 8 Least Squares Approximation

• the LSA problem – an abstract viewpoint; the orthogonality principle; example;

• rephrasing the problem in terms of projection onto a vector space using a basis for the vector space;

• development of the key equation of LSA (this was developed assuming the orthogonality principle);

• detailed study of the key equation;

• defn of the orthogonal complement (this was needed to complete discussion of the key equation); the orthogonal complement of the rowspace is the nullspace and vice-versa;

• left and right inverse of a matrix; relation to rank of a rectangular matrix;

• special cases of LSA: when \( \mathbf{b} \) lies in the columnspace of \( \mathbf{A} \); when \( \mathbf{b} \) is perp to the columnspace; when the columns of \( \mathbf{A} \) are pairwise orthogonal;

Lec 7 Pairs of subspaces

• change of coordinates via Gauss-Jordan method;

• pairs of subspaces; the intersection of two spaces is always a subspace; the union is not in general;

• defn of the sumspace; this is always a subspace; relating the dimension of the sumspace to the dimension of the intersection;

• geometric notions: length via the Pythagoras theorem;

• properties of the length fn;

• notion of perpendicular, again via Pythagoras theorem;

• notion of projection; formula for projecting one vector onto another; the orthogonality principle;
• notion of angle in terms of projections; the Cauchy-Schwartz inequality interpreted in terms of cosine of an angle;

• triangle inequality;