Bring 1 8½ by 11 sheet with whatever you wish on it.

cl.exe bfd66-c-0.exe

The Blackman window

\[ W_{\text{Blackman}} = 0.425 \cdot 5 \cos \left( \frac{2\pi n}{N-1} \right) + 0.08 \cos \left( \frac{4\pi n}{N-1} \right) \]

\[ 0 \leq n \leq N-1 \]
The width of the main lobe for this window is $\frac{12\pi}{N}$ and the main side lobe is 57 dB down from the peak.

**Frequency Measurement for a Single Sinusoid.**

Let $x(t) = A e^{j \omega_0 t + \phi_0}$

Then the sampled version of $x(t)$ is $x(n) = A e^{j \omega_0 n T_s + \phi_0}$

We assume that $x(t)$ has been
Blackman Window
Sampled at Nyquist rate.

The Fourier transform of the signal is:

\[
\mathcal{X}(e^{j\omega}) = A e^{j\phi_0} \sum_{n=-\infty}^{\infty} e^{-j(\omega - \omega_0)nT_s} + j\phi_0
\]

Assume that we collect \(N\) samples of the sequence

\[
x(n)
\]

\[
\mathcal{X}(e^{j\omega}) = A e^{j\phi_0} \sum_{n=0}^{N-1} e^{-j((\omega - \omega_0)T_s/2)(N-1) - \phi_0}
\]

\[
= A e^{-j((\omega - \omega_0)T_s/2(N-1) - \phi_0)}
\]

\[
= A e^{\frac{\sin((\omega - \omega_0)NT_s/2)}{\sin((\omega - \omega_0)T/2)}
\]
\[ Ae \sum_{n=0}^{N-1} e^{-j(w-w_0) n T_s} \]

\[ = Ae^j \phi_0 \frac{1 - e^{-j(w-w_0) N T_s}}{1 - e^{-j(w-w_0) T_s}} \]

\[ = Ae^j \phi_0 \frac{e^{-j(w-w_0) N T_s/2} \sin((w-w_0) N T_s/2)}{\sin((w-w_0) T_s/2)} \]

\[ = Ae^{j(\phi_0 - (w-w_0)(N-1)T_s/2)} \frac{\sin((w-w_0) N T_s/2)}{\sin((w-w_0) T_s/2)} \]
Let us evaluate $X(e^{j\omega})$

at $\omega = \omega_0$

$X(e^{j\omega_0 T_s}) = AN e^{j\phi_0}$

$|X(e^{j\omega_0 T_s})| = AN$

But since $\frac{\sin \omega T_s N/2}{\sin \omega T_s / 2} \leq N$

At $\omega \neq 0$. The point $\omega = \omega_0$ is the global maximum. We thus look for the point at which $X(e^{j\omega T_s})$ is maximum.
This point corresponds to the sinusoid $Ae^{j\omega_0 t + \phi_0}$.

In practice, we find or compute DFT using DFT. In this case, it may not be possible to find the global maximum, since in general the points at which the spectrum is sampled may not correspond to 0.
In this case, we look for an index \( k_0 \) for which \( X(k) \) is maximum. However, we can enhance our measurement either by zero padding or techniques such as zoom FFT.

**Frequency Measurement of 2 complex sinusoids.**

Let \( x(t) = A_1 e^{j(\omega_1 t + \phi_1)} + A_2 e^{j(\omega_2 t + \phi_2)} \)
Let us sample \( x(t) \) at Nyquist-rate:

\[
x(n) = A_1 e^{j(w_1 n T_s + \Phi_1)} + A_2 e^{j(w_2 n T_s + \Phi_2)}
\]

\[-\pi \leq w_i T_s \leq \pi \quad i = 1, 2\]

Let us collect \( N \) samples from the sequence \( x(n) \), and compute its Fourier transform:

\[
\hat{X}(e^{j\omega T_s}) = A_1 e^{j\Phi_1} \sum_{n=0}^{N-1} e^{-j(w-\omega_1) n T_s} + A_2 e^{j\Phi_2} \sum_{n=0}^{N-1} e^{-j(w-\omega_2) n T_s}
\]
\[
A_1 e^{-j((w-w_1)Ts/2(N-1)-\Phi_1)} \frac{\sin ((w-w_1)TsN/2)}{\sin ((w-w_1)Ts/2)} \\
+ A_2 e^{-j((w-w_2)Ts/2(N-1)-\Phi_2)} \frac{\sin ((w-w_2)TsN/2)}{\sin ((w-w_2)Ts/2)}
\]

Let us evaluate \( \hat{X}(e^{-j\omega_1 Ts}) \)

at \( w = w_1 \), then

\[
\hat{X}(e^{j\omega_1 Ts}) = NA_1 e^{j\Phi_1} + A_2 e^{-j((w_1-w_2)Ts/2(N-1)-\Phi_2)} \frac{\sin ((w_1-w_2)TsN/2)}{\sin ((w_1-w_2)Ts/2)}
\]
If $A_2$ is zero then
\[ |z(e^{j\omega_1}T_s)| = NA_1 \]
else if $A_2 \neq 0$ then we note that
\[
\frac{\sin((\omega_1-\omega_2)T_s)}{\sin((\omega_1-\omega_2)T_s/2)} \ll N
\]
However, if \( w_1 \) is close
to \( w_2 \), then \( \frac{\sin (w_1-w_2) T s/2}{\sin (w_1-w_2) T s/2} \)
is closer to \( N \), and as
a result we may not be
able to correctly identify
the sinusoid at \( w_1 \). Also if
\( A_2 \gg A_1 \), we may not be
able to identify the sinusoid
at \( w_1 \).