HW: use the .wav file on the class website and pass it through the filter you designed using the Hamming window of length 63. Plot the spectrum before and after filtering. Listen to the result. Hand in your program, and do not use Matlab's built-in functions (filt)
\[ y(n) = \sum_{k=0}^{N=62} x(k) h(n-k) \]

Optimality of the Impulse response truncation method.

\[ H_D(e^{j\omega}) \approx \text{Desired filter response} \]

\[ H(e^{j\omega}) \approx \text{Designed filter}. \]

Then the Mean Squared Error (MSE) of the designed filter is

\[ \Sigma^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |H_D(e^{j\omega}) - H(e^{j\omega})|^2 d\omega \]

Note: Note that by the Parseval's
Theorem we have

\[ \sum_{n=-\infty}^{\infty} \left( h_D(n) - h(n) \right)^2 \]

\[ = \sum_{n=-\infty}^{-1} h_D(n) + \sum_{n=N+1}^{\infty} h_D^2(n) + \sum_{n=0}^{N} \left( h_D(n) - h(n) \right)^2 \]

\( (N+1) \) is the length of the filter

The first 2 terms are independent of our design, and thus, in order to minimize \( \sum^2 \), we need to minimize the third term, which is
always positive. Thus $\xi^2$
is minimum when the third

\[ \sum_{n=0}^{N} (h_0(n) - h(n))^2 = 0 \]
is zero.

Designs based on windowing

techniques.

For all filters designed using
the windowing technique, the
stop band and pass band ripples
are equal. (Gibbs Phenomenon)
The size of the ripples is independent of the order of the filter and only depends on the type of window used. The width of the transition band depends on the order of the window (filter), and its type.

Designs based on Rectangular window
$\delta_p \triangleq$ peak pass band ripple.

$\delta_s \triangleq$ peak stop band ripple

$\delta_s = \delta_p = 0.09$

Width of the transition band is $\frac{4\pi}{L}$ (L is the length of the window).

Designs based on the Bartlett window
for this window

\[ 8\delta = \delta_s = 0.05 \]

width of the transition band

\[ \frac{8\pi}{L} \]

\[ \begin{array}{c}
0.05 \\
0.95
\end{array} \]
\[ \begin{array}{c}
1.45 \text{ dB} \\
-26 \text{ dB}
\end{array} \]

As can be seen the ripples are slightly smaller while the transition band is twice as large. This window is usually used for filter design.
Designs based on the Hann window.

For this window

\[ \delta p = \delta s = 0.0063 \]

The width of the transition band is slightly less than

\[ \frac{8\pi}{L} \]

0.9937

0.0063

-44 dB

\[ \frac{8\pi}{L} \]
Designs based on Hamming window.

For this window

\[ 8p = 8_\pi = 0.0022 \]

Width of the transition band is slightly less than \[ \frac{8\pi}{T} \].
The Hamming window is frequently used for filter design. The ripples are small and the width of transition band can be made very small by increasing the filter order. Designs based on the Blackman window.

For this window $S_p = S_s = 0.0002$ and the width of the transition band
\[ R = \frac{12\pi}{L} \]

Designs based on the Kaiser window

\[ \delta P = 55.0000 \]

Let \( \alpha = 10 \) then

\[ \text{Width of the transition band} \approx \frac{13\pi}{L} \]
Therefore, if we increase the filter length, we can indeed design very good filters using the Kaiser window.