Cascaded Realization

Note that $H(z)$ can be written:

$$H(z) = b_0 \prod_{i=1}^{\frac{N}{2}} \frac{(1 + h_i z^{-1} + h_{i+1} z^{-2})}{(1 + g_i z^{-1} + g_{i+1} z^{-2})}$$

$$= b_0 \prod_{i=1}^{\frac{N}{2}} H_i(z)$$

$H_i(z) \equiv Bi$-quad.
Direct Realization of FIR Filters.

FIR filters usually have linear phase.

\[ y(n) = \sum_{m=0}^{N-1} h(m) x(n-m) \]

If \( N \) is even then

\[ y(n) = \sum_{m=0}^{\frac{N}{2}-1} h(m) \left[ x(n-m) + x(n-N+m+1) \right] \]

If \( N \) is odd

\[ y(n) = h\left(\frac{N-1}{2}\right) x\left(\frac{N-1}{2}\right) + \sum_{m=0}^{\frac{N-3}{2}} h(m) \left[ x(n-m) + x(n-N+m+1) \right] \]
Adaptive Filters (Equalization)

The primary applications of filtering in communication systems are to select the desired signals, minimize the effects of noise and interference, modify the spectra of signals, and shape the time domain properties of digital waveforms. Filters used in communications systems are sometimes required.
to be adaptive. These filters are required to change their response with the properties of the input signal or the channel. For example, a filter designed to remove signal distortion introduced by the channel should change its response as the channel characteristics change. The most commonly used adaptive filter structure is the tapped...
delay line (TDL). In many communication systems, the channel response is only partially known and is often time varying. In order to compensate for these changes, one can conceive of an equalizing filter at the receiver with transfer function

\[ H_{eq}(w,t) = \frac{1}{H_c(w,t)} \]

\[ H_{eq}(w,t) \cdot H_c(w,t) = 1 \]
Or if we are dealing with a sampled time sequence (i.e., the output of the channel is sampled at times \( nT \)), then
\[
H(e^{j\omega}nT) = (H(e^{j\omega}nT))^\dagger
\]

Tapped delay line (TDL)

\[
X(n+m)\quad \times \quad C_{-M} \quad \times \quad C_{-M+1} \quad \times \quad \ldots \quad \times \quad C_{M+3} \quad \times \quad C_{M-1} \quad \times \quad C_{M} \quad \rightarrow \quad y(n)
\]
Note this is a FIR structure and note that

\[ H_{\text{eq}}(e^{j\omega}) = \frac{Y(e^{j\omega})}{Y(e^{j\omega})} \]

\[ = \sum_{k=-m}^{m} c_k e^{-j\omega k} \]

One of the best features of the TDZ structure is the relative ease with which it
can be modified, to compensate either for the time varying channel, or the lack of complete knowledge about such characteristics. Note that if the tap values are changed, the transfer function changes accordingly.

A very reasonable way to implement the adaptive equalizer (filter) is to vary the Creff (gains) So as
to minimize the mean square error between the actual equalizer output and the desired output. (Ideal output).

Of course, in reality, unless we use a training sequence, the receiver does not know the ideal output. Of course, but under certain conditions one can
assume that the output is almost known. For example, in the digital case the ideal output would be the error-free digital waveform, input to transmitter. If the channel does not introduce too much noise and distortion, then it is safe to say that the decisions are almost always correct, and hence can be
as ideal output for the equalizer purposes.

\[ \text{Squared error fn.} \]

Error Surface is assumed convex.

In two variable a fn is convex if

\[ f(\frac{x_1 + x_2}{2}) \leq \frac{1}{2} f(x_1) + \frac{1}{2} f(x_2) \]
Let us assume that we would like to choose

\[ \{ c_k \} \simeq \left[ z(n) - y(n) \right]^2 \]

is minimized when \( z(n) \) is the ideal signal, or the desired output, \( y(n) \) is the output of the equalizer.
Initially, let us assume that $x(n)$ (input to the channel) and $y(n)$ are given (known). Commonly used equalization algorithms are based on the gradient technique, and can be derived as follows:

$$e^2(n) = \left[ z(n) - y(n) \right]^2$$

$$= \left[ z(n) - \sum_{k=-m}^{m} c_k x(n-k) \right]^2$$

Now, $$\frac{de^2}{dc_m} =$$
\[
\frac{de^2}{dc_m} = -2 \left[ e(n) - \sum_{k=-m}^{m} c_k k(n-k) \right] x(n-m) \\
-2e(n) x(n-m) \\
\frac{dC_m(n)}{dn} = -\mu \frac{d}{dC_m(n)} e^2(n) \\
\Rightarrow \frac{dC_m(n)}{dn} = 2\mu e(n) x(n-m) \\
\Rightarrow C_m(n+1) - C_m(n) = 2\mu e(n) x(n-m) \\
\Rightarrow \boxed{C_m(n+1) = C_m(n) + 2\mu e(n) x(n-m)}
\]
ADAPTIVE EQUALIZATION

SHAHID QURESHI

High speed data transmission over voice band telephone lines.

INTRODUCTION

The rapidly rising need for higher speed data transmission to furnish computer communications has been met primarily by utilizing the widespread network of voice-bandwidth channels developed for voice communications. A modulator-demodulator (MODEM) is required to carry digital signals over these analog passband (nominally 300 to 3000 Hz) channels by translating binary data to voice-frequency signals and back (Fig. 1).

Real analog channels reproduce at their output a transformed and corrupted version of the input waveform. Statistical corruption of the waveform may be additive and/or multiplicative, because of possible background thermal noise, impulse noise and fades. Examples of deterministic (although not necessarily known) transformations performed by the channel are frequency translation, nonlinear or harmonic distortion and time dispersion.

![Fig. 1. Data transmission system.](image)

In telephone lines, time dispersion results from the deviation of the channel frequency response from the ideal characteristics of constant amplitude and linear phase (or constant delay). The idea of equalization is simply to compensate for nonideal characteristics by additional filtering, and dates back to the use of loading coils to improve the characteristics of telephone cables for voice transmission.

A modem transmitter collects an integral number of bits of data at a time and encodes them into symbols for transmission at the signaling rate. In pulse amplitude modulation, each signal is a pulse whose amplitude level is determined by the symbol, e.g., amplitudes of −3, −1, 1, and 3 for quaternary transmission. Efficient digital communication systems the effect of each symbol transmitted over a time dispersive channel extends beyond the time interval used to represent that symbol. The distortion caused by the resulting overlap of received symbols is called intersymbol interference (ISI) [1]. This distortion is one of the major obstacles to reliable high speed data transmission over low-background-noise channels of limited bandwidth.

It was recognized early in the quest for high speed data transmission that rather precise compensation, or equalization, is required to reduce the intersymbol interference introduced by the channel. In addition, in most practical situations the channel characteristics are not known beforehand. For medium-speed (up to 2400 b/s) modems it is usually adequate to design and use a compromise (or statistical) equalizer which compensates for the average of the range of expected channel amplitude and delay characteristics. However, the variation in the characteristics within a class of channels, as in the lines found in the switched telephone network, is large enough so that automatic adaptive equalization is used nearly universally for speeds higher than 2400 b/s. Even 2400 b/s modems now often incorporate this feature.

INTERSYMBOL INTERFERENCE

Intersymbol interference arises in all pulse-modulation systems, including frequency-shift keying (FSK), phase-shift keying (PSK) and quadrature amplitude modulation (QAM). However, its effect can be most easily described for a baseband pulse-amplitude modulation (PAM) system. A model of such a PAM communication system is shown in Fig. 2. A baseband equivalent model such as this can be derived for any linear modulation scheme. In this model the "channel" includes the effects of the transmitter filter, the modulator, the transmission medium and the demodulator.

A symbol $x_n$, one of L discrete amplitude levels, is transmitted at instant $mt$ through the channel, where $T$ seconds is the signaling interval. The channel impulse response $h(t)$ is shown in Fig. 3. The received signal $r(t)$ is the superposition of the impulse responses of the channel to each transmitted symbol and additive white Gaussian noise $n(t)$:

$$r(t) = \sum_n x_n h(t - nT) + n(t).$$

If we sample the received signal at instant $kT + t_0$, where $t_0$
accounts for the channel delay and sampler phase, we obtain
\[ r(t_0 + kT) = x_k h(t_0) + \sum_{j \neq k} x_j h(t_0 + kT - jT) + n(t_0 + kT). \]

The first term on the right is the desired signal since it can be used to identify the transmitted amplitude level. The last term is the additive noise, while the middle sum is the interference from neighboring symbols. Each interference term is proportional to a sample of the channel impulse response, \( h(t_0 + iT) \), spaced a multiple \( IT \) of symbol intervals \( T \) away from \( t_0 \) as shown in Fig. 3. The ISI is zero if and only if \( h(t_0 + iT) = 0, i \neq 0 \); that is, if the channel impulse response has zero crossings at \( T \)-spaced intervals.

When the impulse response has such uniformly-spaced zero crossings, it is said to satisfy Nyquist's first criterion. In frequency domain terms, this condition is equivalent to

\[ H(f) = \text{constant for } |f| < 1/2T. \]

\( H(f) \) is the channel frequency response and \( H(f) \) is the "folded" (altered or overlapped) channel spectral response after symbol-rate sampling. The band \( |f| < 1/2T \) is commonly referred to as the Nyquist or minimum bandwidth. When \( H(f) = 0 \) for \( |f| > 1/T \) (the channel has no response beyond twice the Nyquist bandwidth), the folded response \( H(f) \) has the simple form

\[ H(f) = H(f) + H(-1/f), 0 \leq f \leq 1/T. \]

Figures 4 (a) and (d) show the amplitude response of two linear-phase lowpass filters: one an ideal filter with Nyquist bandwidth and the other with odd (or vestigial) symmetry around 1/2T Hz. As illustrated in Fig. 4 (b) and (c), the folded spectrum of each filter satisfies Nyquist's first criterion. One class of linear-phase filters, commonly referred to in the literature [1], is the raised-cosine family with cosine rolloff around 1/2T Hz.

In practice, the effect of ISI can be seen from a trace of the received signal on an oscilloscope with its time base synchronized to the symbol rate. Figure 5 shows a trace (eye pattern) for a two-level or binary PAM system. If the channel satisfies the zero ISI condition, there are only two distinct levels at the sampling time. The eye is then fully open and the peak distortion is zero. Peak distortion (Fig. 5) is the ISI that occurs when the data pattern is such that all intersymbol interference terms add to produce the maximum deviation from the desired signal at the sampling time.

The purpose of an equalizer, placed in the path of the received signal, is to reduce the ISI as much as possible to maximize the probability of correct decisions.

**Linear Transversal Equalizers**

Among the many structures used for equalization the simplest is the transversal (tapped delay-line or nonrecursive) equalizer shown in Fig. 6. In such an equalizer the current and past values \( r(t - nT) \) of the received signal are linearly weighted by equalizer coefficients (tap gains) \( e_n \) and summed to produce the output. If the delays and tap-gain multipliers are analog, the continuous output of the equalizer \( x(t) \) is
sampled at the symbol rate and the samples go to the decision device. In the new universally used digital implementation, samples of the received signal at the symbol rate are stored in a digital shift register (or memory), and the equalizer output samples (sums of products) $z(t_0 + kT)$ or $z_t$ are computed digitally, once per symbol, according to

$$z_t = \sum_{n=0}^{N-1} c_n r(t_0 + kT - nt)$$

where $N$ is the number of equalizer coefficients.

The equalizer coefficients, $c_n, n = 0, 1, \ldots, N - 1$ may be chosen to force the samples of the combined channel and equalizer impulse response to zero at all but one of the $N$ T-spaced instants in the span of the equalizer. This is shown graphically in Fig. 7. Such an equalizer is called a zero-forcing (ZF) equalizer [2].

If we let the number of coefficients of a ZF equalizer increase without bound, we would obtain an infinite-length equalizer with zero ISI at its output. The frequency response $C(f)$ of such an equalizer is periodic, with a period equal to the symbol rate $1/T$. The frequency response $H(f)$ is determined by the folded frequency response $H(f)$. The combined response of the channel, in tandem with the equalizer, must satisfy the zero ISI condition or Nyquist's first criterion,

$$C(f) H^*(f) = 1 \quad |f| \leq 1/2T.$$  

From the above expression we see that an infinite-length zero-ISI equalizer is simply an inverse filter, which inverts the folded frequency response of the channel. A finite-length ZF equalizer approximates this inverse and so may excessively enhance noise at frequencies where the folded channel spectrum has high attenuation.

Clearly, the ZF criterion neglects the effect of noise altogether. Also, a finite-length ZF equalizer is guaranteed to minimize the peak distortion or worst-case ISI only if the peak distortion before equalization is less than 100 percent; i.e., if a binary eye is initially open. However, at high speeds on bad channels this condition is often not met.

The least mean-square (LMS) equalizer [1] is more robust. Here the equalizer coefficients are chosen to minimize the mean-square error—the sum of squares of all the ISI terms plus the noise power at the output of the equalizer. Therefore, the LMS equalizer maximizes the signal-to-distortion ratio at the equalizer output within the constraints of the equalizer length and delay.

The delay introduced by the equalizer depends on the position of the main or reference tap of the equalizer. Typically, the tap gain corresponding to the main tap is the largest.

If the values of the channel impulse response at the sampling instants are known, the $N$ coefficients of the ZF and the LMS equalizers can be obtained by solving a set of $N$ linear simultaneous equations for each case.

**AUTOMATIC SYNTHESIS**

Before regular data transmission begins, automatic synthesis of the ZF or LMS equalizers for unknown channels, which involves the iterative solution of one of the above-mentioned sets of simultaneous equations, should be carried out during a training period.

Most current high-speed modems use LMS equalizers because they are more robust and superior to the ZF equalizers in their convergence properties. In the remainder of this article we shall restrict our attention to LMS equalizers.

During the training period, a known signal is transmitted and a synchronized version of this signal is generated in the receiver to acquire information about the channel characteristics. The training signal may consist of periodic isolated pulses or a continuous sequence with a broad, even spectrum such as the widely used maximum-length shift-register or pseudo-noise (PN) sequence [1,3]. The latter has the advantage of much greater average power, and hence a larger received signal-to-noise ratio (SNR) for the same peak transmitted power. The training sequence must be at least as long as the length of the equalizer so that the transmitted signal spectrum is adequately dense in the channel bandwidth to be equalized.

![Fig. 7. Combined impulse response of a channel and zero-forcing equalizer in tandem.](image)

![Fig. 8. Automatic adaptive equalizer.](image)
Given a synchronized version of the known training signal, a sequence of error signals \( e_n = a_n - z_n \) can be computed at the equalizer output (Fig. 8), and used to adjust the equalizer coefficients to reduce the sum of the squared errors. The most popular equalizer adjustment method involves updates to each tap gain during each symbol interval. Iterative solution of the coefficients of the equalizer is possible because the mean-square error (MSE) is a quadratic function of the coefficients. The MSE may be envisioned as an N-dimensional paraboloid (funnel bowl) with a bottom or minimum. The adjustment to each tap gain is in a direction opposite to an estimate of the gradient of the MSE with respect to that tap gain. The idea is to move the set of equalizer coefficients closer to the unique optimum set corresponding to the minimum MSE. This symbol-by-symbol procedure is commonly referred to as the continual or stochastic update method because, instead of the true gradient of the mean-square error,

\[
\frac{\partial E[e^2]}{\partial c_n (k)}
\]

a noisy but unbiased estimate

\[
\frac{\partial e^2}{\partial c_n (k)} = 2 e_n r_i (v_0 + k \tau - n \tau)
\]

is used. Thus, the tap gains are updated according to

\[
c_{n+1}(k) = c_n(k) - \Delta c_n r_i (v_0 + k \tau - n \tau), \quad n = 0, 1, \ldots, N-1,
\]

where \( c_n(k) \) is the nth tap gain at time \( k \), \( e_n \) is the error signal and \( \Delta \) is a positive adaptation constant or step size.

**EQUALIZER CONVERGENCE**

The convergence behavior of the stochastic update method is hard to analyze. However, for a small step size and a large number of iterations, the behavior is similar to the steepest-descent algorithm, which uses the actual gradient rather than a noisy estimate.

Here we list some general convergence properties: (a) fastest convergence (or shortest settling time) is obtained when the (folded) power spectrum of the symbol-rate sampled equalizer input is flat, and when the step size \( \Delta \) is chosen to be the inverse of the product of the received signal power and the number of equalizer coefficients; (b) the larger the variation in the above-mentioned folded power spectrum, the smaller the step size must be, and therefore the slower the rate of convergence; (c) for systems where sampling causes aliasing (channel foldover or spectral overlap), the convergence rate is affected by the channel delay characteristics and the sampler phase, because they affect the aliasing. This will be explained more fully later.

**ADAPTIVE EQUALIZATION**

After the initial training period, the coefficients of an adaptive equalizer may be continually adjusted in a decision-directed manner. In this mode the error signal \( e_n = a_n - z_n \) is derived from the final (not necessarily correct) receiver estimate \( \hat{x}_n \) of the transmitted sequence \( \{x_n\} \). In normal operation the receiver decisions are correct with high probability, so that the error estimates are correct often enough to allow the adaptive equalizer to maintain precise equalization. Moreover, a decision-directed adaptive equalizer can track slow variations in the channel characteristics or linear perturbations in the receiver front end, such as slow jitter in the sampler phase.

The larger the step size, the faster the equalizer tracking capability. However, a compromise must be made between fast tracking and the excess mean-square error of the equalizer. The excess MSE is that part of the error power in excess of the minimum attainable MSE (with tap gains frozen at their optimum settings). This excess MSE, caused by tap gains wandering around the optimum settings, is directly proportional to the number of equalizer coefficients, the step size and the channel noise power. The step size that provides the fastest convergence results in a mean-square error is, on the average, 3 dB worse than the minimum achievable MSE. In practice, the value of the step size is selected for fast convergence during the training period and then reduced for fine tuning during the steady-state operation (or data mode).

**EQUALIZERS FOR QAM SYSTEMS**

So far we have only discussed equalizers for a baseband PAM system. Modern high-speed modems almost universally use phase-shift keying (PSK) for lower speeds, e.g., 2400 to 4800 b/s, and combined phase and amplitude modulation or, equivalently, quadrature amplitude modulation (QAM) [1], for higher speeds, e.g., 4800 to 9600 or even 14,400 b/s. QAM is as efficient in bits/second per Hz as vestigial- or single-sideband modulation—yet enables a coherent carrier to be derived and phase jitter to be tracked using easily implemented decision-directed carrier recovery techniques.

Figure 9 shows a generic QAM system, which may also be used to implement PSK or combined amplitude and phase modulation. Two double-sideband suppressed-carrier AM signals are superimposed on each other at the transmitter and separated at the receiver, using quadrature or orthogonal carriers for modulation and demodulation. It is convenient to represent the in-phase and quadrature channel lowpass filter output signals in Fig. 9 by \( y_1(t) \) and \( y_2(t) \), as the real and imaginary parts of a complex-valued signal \( y(t) \). (Note that the signals are real, but it will be convenient to use complex notation.)

The baseband equalizer [4], with complex coefficients \( c_n \), operates on samples of this complex signal \( y(t) \) and produces complex equalized samples \( z(k) = z_n(k) + j z_n(k) \), as shown in Fig. 10. This figure illustrates more concretely the concept of
a complex equalizer as a set of four real transversal filters (with cross-coupling) for two inputs and two outputs. While the real coefficients \( c_n, n = 0, \ldots, N - 1 \), help to combat the intersymbol interference in the in-phase and quadrature channels, the imaginary coefficients \( c_n, n = 0, \ldots, N - 1 \), counteract the cross interference between the two channels. The latter may be caused by asymmetry in the channel characteristic, around the carrier frequency.

The coefficients are adjusted to minimize the mean of the squared magnitude of the complex error signal, \( e(k) = e_i(k) + j e_q(k) \), where \( e_i \) and \( e_q \) are the differences between \( x_i \) and \( x_q \), and their desired values. The update method is similar to the one used for the PAM equalizer except that all variables are complex-valued;

\[
c_n(k + 1) = c_n(k) - \Delta c_n y^n(r_0 + kT - nT), \quad n = 0, 1, \ldots, N - 1,
\]

where \( y^n \) is the complex conjugate of \( y \). Again, the use of complex notation allows the writing of this single concise equation, rather than two separate equations involving four real multiplications, which is what really has to be implemented.

The complex equalizer can also be used at passband [5] to equalize the received signal before demodulation as shown in Fig. 11. Here the received signal is split into its in-phase and quadrature components by a pair of so-called phase-splitting filters, with identical amplitude responses and phase responses that differ by 90°. The complex passband signal at the output of these filters is sampled at the symbol rate and applied to the equalizer delay line in the same way as at baseband. The complex output of the equalizer is demodulated by multiplication by a complex exponential as shown in Fig. 11, before decisions are made and the complex error computed. Further, the error signal is remodulated before it is used in the equalizer adjustment algorithm. The main advantage of implementing the equalizer in the passband is that the error signal can be fed back for phase correction without delay, thus enabling fast phase jitter to be tracked more effectively. The same advantage can be attained with a baseband equalizer by putting a jitter tracking loop after the equalizer.

**DECISION-FEEDBACK EQUALIZERS**

We have discussed different placements and adjustment methods for the equalizer, but the basic equalizer structure has remained a linear and nonrecursive filter. A simple nonlinear equalizer [6], which is particularly useful for channels with severe amplitude distortion, uses decision feedback to cancel the interference from symbols which have already been detected. Figure 12 shows such a decision-feedback equalizer (DFE). The equalized signal is the sum of the outputs of the forward and feedback parts of the equalizer. The forward part is like the linear transversal equalizer discussed earlier. Decisions made on the equalized signal are fed back via a second transversal filter. The basic idea is that if the value of the symbols already detected are known (past decisions are assumed to be correct), then the ISI contributed by these symbols can be canceled exactly, by subtracting past symbol values with appropriate weighting from the equalizer output. The weights are samples of the tail of the system impulse response including the channel and the forward part of the equalizer.

The forward and feedback coefficients may be adjusted simultaneously to minimize the mean squared error. The update equation for the forward coefficients is the same as for the linear equalizer. The feedback coefficients are adjusted according to

\[
b_m(k + 1) = b_m(k) - \Delta b_m d_{m+1}, \quad m = 1, \ldots, M,
\]

where \( d_k \) is the kth symbol decision, \( b_m(k) \) is the mth feedback coefficient at time \( k \) and there are \( M \) feedback coefficients in all. The optimum LMS settings of \( b_m, m = 1, \ldots, M \), are
those that reduce the ISI to zero, within the span of the feedback part, in a manner similar to a ZF equalizer. Note that since the output of the feedback section of the DFE is a weighted sum of noise-free past decisions, the feedback coefficients play no part in determining the noise power at the equalizer output.

Given the same number of overall coefficients, does a DFE achieve less mean squared error than a linear equalizer? There is no definite answer to this question. The performance of each type of equalizer is influenced by the channel characteristics and sampler phase, as well as the actual number of coefficients and the position of the reference or main tap of the equalizer. However, the DFE can compensate for amplitude distortion without as much noise enhancement as a linear equalizer. The DFE performance is less sensitive to the sampler phase.

An intuitive explanation for these advantages is as follows: The coefficients of a linear transversal equalizer are selected to force the combined channel and equalizer impulse response to approximate a unit pulse. If a DFE, the ability of the feedback section to cancel the ISI, because of a number of the past symbols, allows more freedom in the choice of the coefficients of the forward section. The combined impulse response of the channel and the forward section may have nonzero samples following the main pulse. That is, the forward section of a DFE need not approximate the inverse of the channel characteristics, and so avoids excessive noise enhancement and sensitivity to sampler phase.

When a particular incorrect decision is fed back, the DFE output reflects this error during the next few symbols because the incorrect decision traverses the feedback delay line. Thus, there is a greater likelihood of more incorrect decisions following the first one, i.e., error propagation. Fortunately, the error propagation in a DFE is not catastrophic. On typical channels, errors occur in short bursts that degrade performance only slightly.

FRACTIONALLY-SPACED EQUALIZERS

A fractionally-spaced transversal equalizer [7,8] is shown in Fig. 13. The delay line taps of such an equalizer are spaced at an interval \( r \) which is less than, or a fraction of, the symbol interval \( T \). The tap spacing \( r \) is typically selected such that the bandwidth occupied by the signal at the equalizer input is \( 1/2r \), i.e., \( r \)-spaced sampling satisfies the sampling theorem. In an analog implementation, there is no other restriction on \( r \) and the output of the equalizer can be sampled at the symbol rate. In a digital implementation \( r \) must be \( K/M \), where \( K \) and \( M \) are integers and \( M > K \). In practice, it is more convenient to choose \( r = T/M \), where \( M \) is a small integer, e.g., The received signal is sampled and shifted into the equalizer at \( nT + nT/2 \) and the output is produced each symbol interval (for every 2 input samples):

\[
{x_n} = \sum_{k=0}^{N-1} c_r (t_k + kT - nT/2).
\]

The coefficients of a \( T/2 \) equalizer may be updated once per symbol based on the error computed for that symbol, according to

\[
c_r (k+1) = c_r (k) - \Delta n (t_k + kT - nT/2),
\]

where \( n = 0, 1, \ldots, N - 1 \).

One important property of a fractionally-spaced equalizer (FSE) is the insensitivity of its performance to the choice of sampler phase. This distinction between the conventional T-spaced and fractionally-spaced equalizers can be heuristically explained as follows. First, symbol-rate sampling at the input to a T equalizer causes spectral overlap or aliasing, as explained in connection with Fig. 4. When the phases of the overlapping components match they add constructively, and when the phases are 180° apart they add destructively, which results in the cancellation or reduction of amplitude as shown in Fig. 14. Variation in the sampler phase or time instant corresponds to a variable delay in the signal path; a linear phase component with variable slope is added to the signal spectrum. Thus, changes in the sampler phase strongly influence the effects of aliasing; i.e., they influence the amplitude and delay characteristics in the spectral overlap region of the sampled equalizer input. The minimum MSE achieved by the T equalizer is, therefore, a function of the sampler phase. In particular, when the sampler phase causes cancellation of the band-edge (\( |f| = 1/2T \) Hz) components, the equalizer cannot manipulate the null into a flat spectrum at all, or at least without significant noise enhancement (if the null is a depression rather than a total null).

In contrast, there is no spectral overlap at the input to an FSE. Therefore, such an equalizer can adjust the channel spectrum (amplitude and phase) at the two band-edge regions before symbol-rate sampling (and spectral overlap) at the equalizer output. Thus, the sensitivity of the minimum MSE achieved with a fractionally-spaced equalizer with respect to the sampler phase, is typically far smaller than with a T equalizer.

Another point of view is as follows: It has been shown that the optimum receive filter in a linear modulation system is the cascade of a filter matched to the actual channel, with a transversal T-spaced equalizer. The fractionally-spaced equalizer, by virtue of its sampling rate, can synthesize the
best combination of the characteristics of an adaptive matched filter and a T-spaced equalizer, within the constraints of its length and delay. A T-spaced equalizer, with symbol-rate sampling at its input, cannot perform matched filtering. An FSE can effectively compensate for more severe delay distortion and deal with amplitude distortion with less noise enhancement than a T equalizer.

Comparison of the performance of T and T/2 equalizers for QAM systems operating over representative voice-grade telephone circuits has shown the following additional properties: (a) a T/2 equalizer with the same number of coefficients (half the time span) performs almost as well or better than a T equalizer; (b) a pre-equalizer receive shaping filter is not required with a T/2 equalizer; (c) for channels with severe band-edge delay distortion, the T equalizer performs noticeably worse than a T/2 equalizer regardless of the choice of sampler phase.

OTHER APPLICATIONS

In this section we briefly mention applications of automatic or adaptive equalization in areas other than telephone-line modems.

One such application is generalized automatic channel equalization, where the entire bandwidth of the channel is to be equalized without regard to the modulation scheme or transmission rate to be used on the channel. The tap spacing and input sample rate are selected to satisfy the sampling theorem, and the equalizer output is reproduced at the same rate. During the training mode a known signal is transmitted, which covers the bandwidth to be equalized. The difference between the equalizer output and a synchronized reference training signal is the error signal. The tap gains are adjusted to minimize the mean-square error in a manner similar to that used for an automatic equalizer for synchronous data transmission.

On telephone line circuits the primary cause of intersymbol interference is linear distortion because of imperfect amplitude and group-delay characteristics. In radio and underwater channels, ISI is due to multipath transmission, which may be viewed as transmission through a group of channels with different delays. Adaptive equalizers are capable of correcting for ISI due to multipath in the same way as ISI from linear distortion. One special requirement of equalizers intended for use over radio channels is that they be able to track the time varying channel characteristics typically encountered. The convergence rate of the algorithm employed then becomes important during normal data transmission rather than just during the training period.

Experimental use of fixed transversal equalizers has also been made in digital magnetic recording systems. The recording method employed in such a case must be linear instead of the saturated magnetization normally used. Having linearized the "channel," equalization can be employed to combat intersymbol interference at increased recording densities, using a higher symbol rate or multilevel coding.

Two related areas where the techniques developed for adaptive equalization find application are adaptive filtering for cancellation of noise or an interfering signal, and adaptive channel modeling or identification for echo cancellation in communication circuits for speech or data transmission.

IMPLEMENTATION APPROACHES

One may divide the methods of implementing adaptive equalizers into the following general categories: analog, hardwired digital and programmable digital.

Analog adaptive equalizers, with inductor-capacitor (LC) tapped delay lines and switched ladder attenuators as tap gains, were among the first implementations. The switched attenuators later gave way to field-effect transistors as the variable gain elements. Analog equalizers were soon replaced by digitally implemented equalizers for reduced size and increased accuracy. Recently, however, there is renewed interest in large-scale integrated (LSI) analog implementations based on the charge-coupled device (CCD) technology. Here the equalizer input is sampled but not quantized. The sampled analog values are stored and transferred as charge packets. The variable tap gains are typically stored in digital memory locations and the multiplications between the analog sample values and the digital tap gains take place in analog fashion, as with multiplying digital-to-analog converters. This technology is still in infancy and has yet to find its way into practice. However, it has significant potential in applications where the symbol rates are high enough to make digital implementations impractical or very costly.

The most widespread technology of the last decade for adaptive equalizer implementation may be classified as hardwired digital technology. In such implementations the equalizer input is made available in sampled and quantized form suitable for storage in digital shift registers. The variable tap gains are also stored in shift registers and the formation and accumulation of products takes place in logic circuits connected to perform digital arithmetic. This class of implementations is characterized by the fact that the circuitry is hardwired for the sole purpose of performing the adaptive equalization function with a predetermined structure. Examples include the early units based on metal-oxide semiconductor (MOS) shift registers and transistor-transistor logic (TTL) circuits. Later implementations were based on MOS...
LSI circuits with dramatic savings in space, power dissipation and cost.

The most recent trend in implementing adaptive equalizers is toward programmable digital signal processors. Here, the equalization function is performed in a series of steps or instructions in a microprocessor or a digital computation structure specially configured to efficiently perform the type of digital arithmetic (e.g., multiply and accumulate) required in digital signal processing. The same hardware can then be time-shared to perform functions such as filtering, modulation and demodulation in a modem. Perhaps the greatest advantage of programmable digital technology is its flexibility, which permits sophisticated equalizer structures and training procedures to be implemented with ease.

CONCLUSION

This paper serves as a broad-brush introduction to a rich and mature field. Interested readers will find a wealth of information in the brief list of references given here. More comprehensive lists of references are available in [6, 9 and 10]. Despite the maturity of the field, adaptive equalization is still an area of active interest; a good example is fractionally-spaced and fast-training equalizers and their implementation.

REFERENCES


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