10.9 The selectivity factor is

\[ k = \frac{\omega_p}{\omega_s} = 0.3. \]

For a Chebyshev filter that meets the specifications exactly we have

\[ N = \frac{1}{2 \cosh(1/d)} \]

from which we can solve for \( d \):

\[ d = (\cosh(3\arccosh(1/0.3)))^{-1} = 0.007238. \]

Therefore,

\[ s = d(\delta_s^{-2} - 1)^{1/2} = 0.36186. \]

Also, for a Chebyshev-II filter

\[ \omega_p = \omega_s = 3. \]

We now substitute in the design formulas to get

\[ \frac{1}{N} \cosh(1/\varepsilon) = 0.5803. \]

\[ \sinh(0.5803) = 0.6134, \quad \cosh(0.5803) = 1.1731. \]

\[ s_{13} = -0.9201, \quad s_1 = -1.8402. \]

Finally,

\[ H^*(z) = \frac{18.652}{s + 1.8402(z^2 + 1.8402z + 10.1356)^{1/2}}. \]

10.13 Since the pass-band response has to be monotone, only Butterworth and Chebyshev-II filters are eligible. For the former, \( \omega_p = (2\pi \cdot 19.4456)10^3 \) and \( N = 41 \). For the latter, \( \omega_p = (2\pi \cdot 24)10^3, N = 16 \), and \( \varepsilon = 10^{-4} \).

11.4 Express the transfer function as

\[ H^2(z) = 1 + z^{-1} + \ldots + z^{-(N-1)} = \frac{1 - z^{-N}}{1 - z^{-1}}. \]

Now we can realize \( H^*(z) \) in either of the two direct realizations; see Figures 11.4 and 11.6. We have to substitute

\[ a_1 = -1, \quad a_2 = 0, \quad 2 \leq k \leq N, \quad b_0 = 1, \quad b_k = 0, \quad 1 \leq k \leq N - 1, \quad b_N = -1. \]

The corresponding difference equation is

\[ y[n] = y[n-1] + x[n] - x[n-N]. \]

As we see, there are two additions and no multiplications per time point.

11.5

(a) Let the state variables \( s_1(n), s_2(n) \) be the outputs of the top and bottom delay elements, respectively. Then

\[ \begin{bmatrix} s_1(n+1) \\ s_2(n+1) \end{bmatrix} = \begin{bmatrix} 2 \cos \theta_0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} s_1(n) \\ s_2(n) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} A \delta[n], \quad y[n] = \begin{bmatrix} \cos \theta_0 & -1 \end{bmatrix} \begin{bmatrix} s_1(n) \\ s_2(n) \end{bmatrix} + A \delta[n]. \]

In the z domain,

\[ Y^2(z) = A(C(zI_2 - A)^{-1}B + 1). \]

We have

\[ zI_2 - A = \begin{bmatrix} z & -2 \cos \theta_0 \\ \cos \theta_0 & z \end{bmatrix}, \quad (zI_2 - A)^{-1} = \frac{1}{z^2 - 2\cos \theta_0 z + 1} \begin{bmatrix} z & -1 \\ -1 & z \end{bmatrix}. \]

Therefore,

\[ Y^2(z) = A \frac{z \cos \theta_0 - 1}{z^2 - 2\cos \theta_0 z + 1} + A = A \frac{1 - \cos \theta_0 z^{-1}}{1 - 2\cos \theta_0 z^{-1} + z^{-2}}. \]

We see from Table 7.1 that

\[ y[n] = A \cos(\theta_0 n), \quad n \geq 0. \]

(b) The realization can be used to generate a discrete-time cosine signal having a given frequency \( \theta_0 \). It does not require any explicit cosine operation, only multiplications and additions, so it is very efficient in computations. The frequency can be controlled by setting the constant gain \( \cos \theta_0 \) according to the desired frequency.