3.2 It follows from the result for $X(f)$ that $x[n] = \delta[n]$. Since $e^{-0.02t^2} > 0$ for all $t$, we must have that the sampling of $	ext{sinc}(t)$ is $\delta[n]$. The minimum $T$ for which this happens is $T = 1$.

3.8  

(a) The minimum $N$ is $N = 7$. Taking $N = 6$ is not enough, since then the harmonics $k = 3$ and $k = -3$ would add up after sampling, and $X^5[3], X^5[-3]$ would become ambiguous.

(b) Seven samples are enough, since they would enable us to construct seven equations in seven unknowns as follows:

$$
\sum_{k=-3}^{3} X^5[k] \exp \left( j \frac{2\pi k n T}{T_0} \right) = x(nT), \quad 0 \leq n \leq 6.
$$

These equations can be solved for $X^5[k]$, since the samples $x(nT)$ are known. Note that $T/T_0 = 1/N = 1/7$.

(c) In this case there will be aliasing, as shown in Figure 3.1 (in this figure we define $\theta_0 = 2\pi/5.5$). However, the aliased harmonics do not overlap with any of the other harmonics, so unambiguous computation of the $X^5[k]$ is still possible. Seven samples are again sufficient, and the equations are exactly as before, except that $T/T_0 = 1/5.5$.

3.17  

a. $x(t)$ is periodic; since we sample $x(t)$ at an integer number of times per period, $x[n]$ is periodic. The reconstruction $\hat{x}(t)$ will also be periodic.

b. No – this signal is not band-limited, hence we cannot sample it without aliasing.

c. 

$$
x[n] = \begin{cases} 
0 & n \mod 3 = 0, \\
1 & n \mod 6 = 1, 2, \\
-1 & n \mod 6 = 4, 5.
\end{cases}
$$
\[ X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \]  \hspace{1cm} (43)

\[ = \sum_{m=-\infty}^{\infty} \left( x[6m - 2]e^{-j\omega(6m-2)} + x[6m - 1]e^{-j\omega(6m-1)} + x[6m + 1]e^{-j\omega(6m+1)} + x[6m + 2]e^{-j\omega(6m+2)} \right) \]  \hspace{1cm} (44)

\[ = \sum_{m=-\infty}^{\infty} \left( -e^{-j\omega(6m-2)} - e^{-j\omega(6m-1)} + e^{-j\omega(6m+1)} + e^{-j\omega(6m+2)} \right) \]  \hspace{1cm} (45)

\[ = \sum_{m=-\infty}^{\infty} \left( e^{-j\omega(-2)} - e^{-j\omega(-1)} + e^{-j\omega(+1)} - e^{-j\omega(+2)} \right) e^{-j\omega(6m)} \]  \hspace{1cm} (46)

\[ = (e^{j\omega} - e^{-j\omega} - e^{j2\omega} + e^{-j2\omega}) \sum_{m=-\infty}^{\infty} e^{-j\omega(6m)} \]  \hspace{1cm} (47)

\[ = -2j (\sin \omega + \sin 2\omega) \sum_{m=-\infty}^{\infty} e^{-j\omega(6m)} \]  \hspace{1cm} (48)

\[ = -2j (\sin \omega + \sin 2\omega) \sum_{m=-\infty}^{\infty} e^{-j\omega(6m)} \]  \hspace{1cm} (49)

From Poisson's formula (Porat, eq. 2.51) we know that

\[ \sum_{k=-\infty}^{\infty} \delta \left( \omega - \frac{2\pi k}{T} \right) = T \sum_{n=-\infty}^{\infty} e^{jn\omega T} \]  \hspace{1cm} (50)

Substituting \( T = 6 \) and reversing the order of summation we can rewrite this as

\[ \sum_{m=-\infty}^{\infty} e^{-jm6\omega} = \frac{2\pi}{6} \sum_{k=-\infty}^{\infty} \delta \left( \omega - \frac{2\pi k}{6} \right) \]  \hspace{1cm} (51)

thus

\[ X(e^{j\omega}) = -2j (\sin \omega + \sin 2\omega) \frac{\pi}{3} \sum_{k=-\infty}^{\infty} \delta \left( \omega - \frac{\pi k}{3} \right) \]  \hspace{1cm} (52)

\[ = \frac{j2\pi}{3} \sum_{k=-\infty}^{\infty} \left( \sin \frac{k\pi}{3} + \sin \frac{2k\pi}{3} \right) \delta \left( \omega - \frac{\pi k}{3} \right) \]  \hspace{1cm} (53)

which is a train of weighted pulses.

Passing \( x[n] \) through an ideal reconstructor passes only those terms where \( \omega = \frac{n\pi}{3} \) is less than \( \pi \), or \( n \in \{-2, -1, 0, 1, 2\} \), or

\[ \hat{X}(\Omega) = \frac{j2\pi}{3} \sum_{k=-2}^{2} \left( \sin \frac{k\pi}{3} + \sin \frac{2k\pi}{3} \right) \delta \left( \Omega - \frac{\pi k}{3T} \right) \]  \hspace{1cm} (54)

When \( k = 0 \) in the above summation, both \( \sin \) terms are zero. When \( k = 2 \) or \( k = -2 \), the sin terms sum to zero, e.g.,

\[ \sin \frac{2k\pi}{3} \big|_{k=2} = \sin \frac{4\pi}{3} = -\sin \frac{2\pi}{3} = -\sin \frac{k\pi}{3} \big|_{k=2} \]  \hspace{1cm} (55)

hence the only non-zero summation terms we have are \( k = 1 \) and \( k = -1 \):

\[ \hat{X}(\Omega) = \frac{j2\pi}{3} \sum_{k=-1,1} \left( \sin \frac{k\pi}{3} + \sin \frac{2k\pi}{3} \right) \delta \left( \Omega - \frac{\pi k}{3T} \right) \]  \hspace{1cm} (56)
Noting that \( \sin(-x) = -\sin x \),

\[
X(\Omega) = -\frac{j2\pi}{3} \left( \sin \frac{\pi}{3} + \sin \frac{2\pi}{3} \right) \left[ \delta \left( \Omega - \frac{\pi}{3T} \right) - \delta \left( \Omega + \frac{\pi}{3T} \right) \right] 
\]

(57)

\[
= -\frac{j2\pi}{3} \left( \sqrt{3} \right) \left[ \delta \left( \Omega - \frac{\pi}{3T} \right) - \delta \left( \Omega + \frac{\pi}{3T} \right) \right] 
\]

(58)

\[
= \frac{2\sqrt{3}\pi}{3} j \left[ \delta \left( \Omega + \frac{\pi}{3T} \right) - \delta \left( \Omega - \frac{\pi}{3T} \right) \right] 
\]

(59)

which has the transform

\[
x(t) = \frac{2\sqrt{3}\pi}{3} \sin \left( \frac{\pi t}{3} \right)
\]

(61)

3.22

(a) The sampled signal is

\[ x[n] = 3 \cos(0.25\pi n) + 2 \sin(0.625\pi n). \]

The reconstructor assumes that the relationship between the digital and the analog frequencies is \( \theta = 0.005\omega \). The output of the reconstructor is therefore

\[ \hat{x}(t) = 3 \cos(50\pi n) + 2 \sin(125\pi n). \]

(b) Now the sampled signal is

\[ x[n] = 3 \cos(0.5\pi n) + 2 \sin(1.25\pi n) = 3 \cos(0.5\pi n) - 2 \sin(0.75\pi n). \]

Note that the second term is aliased. The reconstructor assumes that the relationship between the digital and the analog frequencies is \( \theta = 0.0025\omega \). The output of the reconstructor is therefore

\[ \hat{x}(t) = 3 \cos(200\pi n) - 2 \sin(300\pi n). \]
3.35

(a) Since the bands \([3, 5]\) and \([7, 9]\) contain only noise, they may be aliased by sampling without harming the useful information in the band \([5, 7]\). We can therefore treat the signal as if it were band limited to \([4, 8]\), and sample it at a rate \(\omega_{\text{sam}} = 8\). As Figure 1 illustrates, this will leave the range \([5, 7]\) nonaliased. The useful signal can be reconstructed by an ideal band-pass filter

\[
H^f(\omega) = \begin{cases} 
1, & 5 \leq |\omega| \leq 7, \\
0, & \text{otherwise}. 
\end{cases}
\]

![Figure 1](image1)

**Figure 1** Pertaining to Solution 3.35.

3.36

We will assume that \(x(t)\) has a flat spectrum in the range \(3.5 \leq |\omega| \leq 4.5\). The CTFT of \(y(t) = x(t)^2\) will be given by

\[
Y_C(\Omega) = \frac{1}{2\pi} X_C(\Omega) * X_C(\Omega), 
\]

which is shown in Fig. 2. It is clear that using \(\Omega_s = 6\) will completely fill the spectrum, and is the minimum sampling rate we can use.

![Figure 2](image2)

**Figure 2:** \(X(\Omega), Y(\Omega)\) and \(Y(e^{j\omega})\)