AN INTRODUCTION TO THE
DECOMPOSITION OF BOOLEAN FUNCTIONS

EE 552 ADVANCED SWITCHING THEORY AND LOGIC DESIGN

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Introduction

The purpose of Function Decomposition is to reduce the number of variables used in a function through the introduction of a new variable that replaces multiple variables. This can only be done on unique functions that contain repetition.

Types of Functions

There are two types of functions that may be decomposed which may be categorized as disjunctive and non-disjunctive. The difference between these two is that in a disjunctive decomposition each input variable goes to only one subfunction while a non-disjunctive decomposition may have an input variable going to multiple subfunctions. This difference is shown in the notation that is used to write a decomposed function. If \( f(w,x,y,z) = h(g(w,x), y,z) \) then the function is disjunctive. If \( f(w,x,y,z) = h(g(w,x,y), y,z) \) then the function is non-disjunctive.

The function \( f(w,x,y,z) = h(g(w,x), y,z) \) means that there is a function \( h \) that is equal to \( f \) that contain the variables \( g,y \) and \( z \) with \( g \) representing the combination of \( w \) and \( x \) into a single variable.

There are different ways to reduce disjunctive and non-disjunctive decompositions so it is important to be able to identify them.

Decomposition Process

The first step is identifying if the function can be decomposed. This is most easily accomplished by looking at the Karnaugh map of the function to determine if any rows or columns contain the same values. In Figure 1 notice that rows 1 and 3 are the same and rows 2 and 4 are the same. When all of the rows or columns may be categorized into two unique patterns then the function may be decomposed in the form of a simple disjunctive decomposition.

The four columns of Figure 2 do not form two distinct patterns so this function is not a simple disjunctive decomposition but it still may be decomposed as a nondisjunctive decomposition. If you were to break Figure 2 into two k-maps with \( y \) equal to 0 in one and 1 in the other you then both k-maps contain the properties of a simple disjunctive decomposition and may be decomposed.
Simple Disjunctive Decomposition

To determine the function of Figure 1 that represents the simple disjunctive decomposition you need to figure out the form of the resulting function. Since Figure 1 has rows that are similar the function should be of the following form \( f(w, x, y, z) = h(w, x, g(y, z)) \) with \( y \) and \( z \) combining to form function \( g \). The value of \( g \) can be found by arbitrarily numbering the matching rows in Figure 1 with either a one or a zero as shown in Figure 3. In this case \( g(y, z) = y'z + yz' \).

![Figure 3](image)

![Figure 1](image)

After the subfunction is determined the rest of the decomposed function may be found as shown in Figure 4 with \( f(w, x, y, z) = w'g + w'x + xg + wx'g' \). This is much simpler than the original function which was \( f(w, x, y, z) = w'x + w'y'z + xy'z + w'y'z' + wy'z' + wx'y'z + wx'yz' \). Although both functions share the same k-map only the decomposed function is considered a disjunctive decomposition. The function in its original form actually produces a nondisjunctive decomposition when it is implemented through logic gates.

Complex Disjunctive Decomposition

This is a subset of simple disjunctive decomposition. When a function has both its rows and columns containing only two distinct patterns of values. This function can be reduced by substituting in two variables to replace all of the variables. An example is shown in the following figure with the resulting function in the form of \( f(w, x, y, z) = k(g(w, x), h(y, z)) \).

![Figure 5](image)
Nondisjunctive Decomposition

The nondisjunctive decomposition is decomposed by breaking the original function into two functions as shown in Figure 5. Then each subfunction may be decomposed through the same procedure as used in the simple disjunctive decomposition.

The resulting function is in the form of \( h(g(w,x,y),y,z) \). The variable \( y \) is repeated since this is the variable that the function was split on.

\[
\begin{array}{c|c|c|c|c}
0 & 0 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 1 \\
1 & 0 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 \\
\end{array}
\]

\[
\begin{array}{c|c|c|c|c}
0 & 0 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 1 \\
1 & 0 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 \\
\end{array}
\]

\[
\begin{array}{c|c|c|c|c}
0 & 0 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 1 \\
1 & 0 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 \\
\end{array}
\]

\[
\begin{array}{c|c|c|c|c}
0 & 0 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 1 \\
1 & 0 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 \\
\end{array}
\]

Through the rules of Shannon's decomposition \( g' \) and \( g1 \) may be reduced to one variable \( g \).

\[
g = y'(g0) + y(g1) = y'(w+x) + y(w'x+wx')
\]

\[
(0, x, y, z) = y'(f_y) + y(f_z) = y'g + y(g' + g') = y + g'z
\]
Conclusion

The resultant function produced by decomposition is usually much simpler than the original but this comes at the cost of the logic involved to make the subfunction. When you are only dealing with a minimal number of variables, as in the case of the previous examples, it is hardly worth the effort. But, in computers where there may be 32 variables or more and repetitive forms are quite common, this becomes a very effective way to reduce the logic.

References

2) James Ellison. EE552 class notes.