Read: H & L chapter 4.6-4.7 (up to p.159), 5.1

Review:

Variations in Model Forms:
Constraints to be satisfied at equality (artificial-variable).
Negative RHS.
Constraints with opposite inequality signs.
Minimization problems.

Radiation Therapy Example:
The Big-M method.

Radiation Therapy Example (continued):
Tracking the solutions for the Big-M method...
Two-Phase Simplex Method:

**Initialization:** Revise constraints of original problem to obtain an obvious BFS for the *artificial problem*.

**Phase I:** Find a BFS for the *real problem* by minimizing the sum of the artificial variables.

**Phase II:** Find an *optimal solution* for the real problem.
How can you tell if the real problem has no feasible solutions?

How can we model variables that are allowed to be negative?

CASE 1: Variables will a lower bound.

CASE 2: Variables with no lower bounds.
Post-optimality Analysis:

- Re-optimization
- Shadow Prices
- Sensitivity Analysis
**Introduction to the Foundations of the Simplex Method:**

**Definitions:**
- Constraint Boundary Equation
- Hyperplane
- Boundary
- Corner-Point Feasible Solution
- Edges
- Adjacent CPF Solutions

**Properties of CPF Solutions:**

**Property 1:**
(a) If there is exactly 1 optimal solution, it must be a CPFS.
(b) If there are multiple opt solns (and a bounded feasible region), \( \geq 2 \) must be adjacent CPFS.

**Property 2:** There are a finite number of CPFS.

**Property 3:** If a CPFS has no adjacent CPFS that are better, then such a CPFS is guaranteed to be an optimal solution.

**In the Augmented Form:**

Properties also hold for BFS.

Each BFS has \( m \) basic variables, and the rest are nonbasic.
The number of nonbasic variables equals \( n + \# \text{ surplus variables} \).
The basic solution is the augmented CPS whose \( n \) defining equations are indicated by the nonbasic variables.

A BFS is a basic solution where all \( m \) basic variables are nonnegative. A BFS is said to be degenerate if any of the \( m \) variables equals zero.

An adjacent CPF solution is reached by
(1) deleting one constraint boundary from the \( n \) defining boundaries
(2) moving along the edge defined by the remaining \( n - 1 \) boundaries
(3) stopping when the first new boundary is reached.
| Homework 3: | 4.6-2 (a) through (g), 4.6-5, 4.6-15, 5.1-4, 5.1-13 | Due in class | September 24, 2002 |