**Midterm**

21 October 2003

1 hour 15 minutes

(7 pages)

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**Hagrid:** "What's comin' will come, an' we'll meet it when it does."

*Here it is, the midterm. Good luck!* ~ E.C.

Name: _____________________________________________________
Rewnar Sisters is a major movie production company that needs a combination of storage and office space for its various filming projects, including a high profile series on a trio of young magical folk.

Pauline is in charge of making the decisions on the amount of storage space ($x_1$) and office space ($x_2$) to rent for the year. She has to solve the following linear program to determine the optimum values for $x_1$ and $x_2$:

Minimize $W = 100x_1 + 300x_2$

s.t. $x_1 + x_2 \geq 10$

$x_1 + 2x_2 \geq 15$

$6x_1 + x_2 \geq 18$

$x_1 - x_2 \leq 10$

$x_1 \geq 0, x_2 \geq 0$
(a) Solve the problem geometrically to show that $W$ is minimized when $x_1 = 11.33$ and $x_2 = 1.67$.  
[ 15 points ]

(b) Identify the optimal basic feasible solution and its defining equations.  
[ 10 points ]
(c) Standardize the constraints and construct the dual problem. [ 15 points ]

(d) Use part (b) to help you determine the optimal basic feasible solution for the dual problem. [ 10 points ]
(e) Construct the final tableaux of the dual problem. [ 20 points ]
(f) Suppose the first constraint becomes

\[ x_1 + x_2 \geq 10 + q \]

What is the allowable range of \( q \) for the solution in (e) to stay feasible and/or optimal? Please provide the geometrical interpretation of the allowable change to stay feasible/optimal using Pauline’s original problem.

[ 20 points ]
(g) Suppose a new constraint is introduced in the original problem. 

\[ x_2 \leq l \]

For what values of \( l \) would the current optimal solution stay feasible and optimal? 

[ 10 points ]

(h) What is an anagram? Please give an example. 

[ 2.5 points ]