Ron: There you go, Harry! You weren’t being thick after all – you were just showing your moral fibre!

*Please show your moral (and intellectual) fibre in this quiz. Good luck! ~ E.C.*
The SWISHER Co. produces two types of brooms, the Cleansweep sports broom and the Bluebottle, a family broom with an anti-burgler buzzer.

The company has three plants. Plant 1 assembles the brooms, Plant 2 installs the anti-burgler buzzer, Plant 3 adds the braking spell (a necessary component to every broom). The profit on each batch of Bluebottle brooms is 10 Galleons, and that for each batch of Cleansweep brooms is 20 Galleons.

A muggle OR team has created the following Linear Programming model to help the company make some executive decisions on the number of batches of each type of broom \( x_1 \) for Bluebottles and \( x_2 \) for Cleansweeps) to be made per cycle:

\[
\begin{align*}
\text{Max} & \quad 10x_1 + 20x_2 \\
\text{s.t.} & \quad 3x_1 + 4x_2 \leq 31 \\
& \quad x_1 \leq 5 \\
& \quad -5x_1 + 6x_2 \leq 18 \\
& \quad x_1 \geq 0, \quad x_2 \geq 0
\end{align*}
\]
(a) Show graphically that the solution to the problem is (3, 5.5) with $Z = 140$. Label ALL the corner point solutions, identify the feasible region and show clearly the profit line that runs through the optimal solution.

(25 points)
(b) Suppose that a very muddled muggle made a mistake and mislabeled the variables in the constraints so that $x_1$ was actually the batches of Cleansweep and $x_2$ the batches of Bluebottles (and plant 2 actually installs the anti-jinx varnish, a feature of the Cleansweep brooms). A rather bright young muggle pointed out that nothing needed to be changed in the program except for the objective, which should now be $20x_1 + 10x_2$. Please determine (graphically) if the optimal solution has changed and the new value of $Z$. 

(20 points)
(c) Suppose that the original model was correct. Propose an objective function for which the problem has multiple solutions. Be sure to graph the new objective function and to point out the optimal solutions on your graph.

(20 points)
(d) Suppose the constraint

\[ 3 \ x_1 \ + \ 4 \ x_2 \leq 31 \]

needs to be satisfied at equality:

\[ 3 \ x_1 \ + \ 4 \ x_2 \ = \ 31 \]

Use the Big M method to solve the problem:

\[
\begin{align*}
\text{Max} \quad & 10 \ x_1 \ + \ 20 \ x_2 \\
\text{s.t.} \quad & 3 \ x_1 \ + \ 4 \ x_2 \ = \ 31 \\
\quad & x_1 \leq 5 \\
\quad & -5 \ x_1 \ + \ 6 \ x_2 \leq 18 \\
\quad & x_1 \geq 0 \ , \ x_2 \geq 0
\end{align*}
\]  

(35 points)
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(e) Name the enchanted winged object that the Seeker in a quidditch team must catch to win 150 points.

(5 points)