THE SIMPLEX METHOD

What is the standard form of an LP?
How to derive the solution graphically?
What is a Corner-Point Feasible Soln / Basic Feasible Soln?

Method:

Initialize: Introduce slack variables, which are the initial basic variables. Decision variables are initial nonbasic variables.

Optimality Test: BFS is optimal iff every coefficient in row 0 is nonnegative. If so, we are done, otherwise continue.

Iteration: Determine entering basic variable – having the most negative coefficient in row 0. This is the pivot column. Determine leaving basic variable by applying the minimum ratio test. Solve for new BFS using elementary row operations.

Variations in Model Forms:

Equality constraints.
Negative RHS.
Constraints with opposite inequality signs.
Minimization problems.

The Big-M method.

The Two-Phase Simplex Method.

I: Find a BFS for the real problem by min the sum of the artificial var.
II: Find an optimal solution for the real problem.

How can you tell if the real problem has no feasible solutions?
How can we model variables that are allowed to be negative?

CASE 1: Variables with a lower bound.
CASE 2: Variables with no lower bounds.
### REVISED SIMPLEX METHOD:

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Basic Var</th>
<th>Coeff</th>
<th>slack</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Z</td>
<td>Z</td>
<td>– c</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>x_B</td>
<td>0</td>
<td>A</td>
<td>1</td>
</tr>
<tr>
<td>Any</td>
<td>Z</td>
<td>1</td>
<td>c_B(B^(-1)A - c)</td>
<td>c_B(B^(-1)b)</td>
</tr>
<tr>
<td></td>
<td>x_B</td>
<td>0</td>
<td>B^(-1)A</td>
<td>B^(-1)</td>
</tr>
</tbody>
</table>

### COMPLEMENTARITY

**Complementary basic solutions property:** Each basic solution in the primal problem has a complementary basic solution in the dual problem where Z = W.

**Complementary slackness property:** The variables in the primal basic solution and the complementary dual basic solution satisfy following rules:

<table>
<thead>
<tr>
<th>Primal variable</th>
<th>Associated Dual variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic</td>
<td>Non-basic (m variables)</td>
</tr>
<tr>
<td>Non-basic</td>
<td>Basic (n variables)</td>
</tr>
<tr>
<td>Decision var x_j</td>
<td>Surplus variable z_j - c_j</td>
</tr>
<tr>
<td>Slack var x_{n+i}</td>
<td>Decision variable y_i</td>
</tr>
</tbody>
</table>

**Complementary optimal basic solutions property:** Each optimal basic solution in the primal problem has a complementary optimal basic solution in the dual problem, where their Z = W.

### DUALITY

**Shortcut for conversion between primal and dual:**

<table>
<thead>
<tr>
<th>Primal</th>
<th>Dual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximize Z</td>
<td>Minimize W</td>
</tr>
<tr>
<td>Constraint i:</td>
<td>Variable y_i:</td>
</tr>
<tr>
<td>≤ form</td>
<td>y_i ≥ 0</td>
</tr>
<tr>
<td>= form</td>
<td>unconstrained</td>
</tr>
<tr>
<td>≥ form</td>
<td>y'_i ≤ 0</td>
</tr>
<tr>
<td>Variable x_j:</td>
<td>Constraint j:</td>
</tr>
<tr>
<td>x_j ≥ 0</td>
<td>≥ form</td>
</tr>
<tr>
<td>unconstrained</td>
<td>unconstrained</td>
</tr>
<tr>
<td>x'_j ≤ 0</td>
<td>≤ form</td>
</tr>
</tbody>
</table>
SENSITIVITY ANALYSIS

Definition: Sensitivity Analysis deals with the effect on the optimal solution of making changes in the values of the model parameters $a_{ij}$, $b_i$, $c_j$.

CASE 1: Changes in $b_i$

One change in the RHS
What is the allowable range to stay feasible?

Simultaneous changes on the RHS
Rule of thumb: Calculate the percentage of the allowable change for each change. If the sum of the percentage changes does not exceed 100%, the shadow prices will still be valid.

CASE 2a: Changes in the Coefficients of a Nonbasic Variable

One change in the Objective Function Coefficient
How to calculate the allowable range to stay optimal for $c_j$.

Simultaneous changes in the Objective Function Coefficients
Rule of thumb: The 100% rule as above.

CASE 2b: Introducing a New Variable

CASE 3: Changes in the Coefficients of a Basic Variable
- Calculate the revised final tableaux.
- Convert this to standard form, and find the new optimal solution.
- Find allowable range to stay optimal?

CASE 4: Introducing new constraints
- What is the graphical effect of this new constraint?
- Does the previous optimal solution violate this constraint?
- What is the optimal solution?

Parametric Approach to Sensitivity Analysis

Investigate the effect of varying individual $b_i$ parameters.
Multiple $b_i$ parameters can be varied at the same time.
**Shamelessly lifted from the HARRY POTTER LEXICON**

**Hogwarts School of Witchcraft and Wizardry**

Students at Hogwarts are grouped into four houses: Gryffindor, Ravenclaw, Hufflepuff, and Slytherin. When the students arrive on September 1 of their first year, they travel from the train station in Hogsmeade to Hogwarts castle by means of a fleet of small boats, led by Hagrid. They are brought into the Entrance Hall, where they are met by McGonagall, who explains the house system:

"The start-of-term banquet will begin shortly, but before you take your seats in the Great Hall, you will be sorted into your houses. The Sorting is a very important ceremony because, while you are here, your house will be something like your family within Hogwarts. You will have classes with the rest of your house, sleep in your house dormitory, and spend free time in your house common room.

"The four houses are called Gryffindor, Hufflepuff, Ravenclaw, and Slytherin. Each house has its own noble history and each has produced outstanding witches and wizards. While you are at Hogwarts, your triumphs will earn your house points, while any rulebreaking will lose house points. At the end of the year, the house with the most points is awarded the house cup, a great honor. I hope each of you will be a credit to whichever house becomes yours."

The students are then led into the Great Hall. There they are sorted into their houses by placing the Sorting Hat onto their heads, whereupon it decides which house they should belong to.

**Quidditch**

Quidditch is played up on broomsticks up in the air. There are four goal posts at either ends of a field. That field is called a Quidditch Pitch. Quidditch has three balls. The ball that scores the points is the Quaffle. The Quaffle is 12 inches in diameter and is made of leather bindings. The Quaffle has made some different changes over the years. The Bludger is probably the most dangerous ball of all of them. It flies through the air being hit by players called beaters. Serious injuries have been caused by Bludgers hitting people and causing them to fall off their brooms. The third and most important ball is the Golden Snitch. The Golden Snitch is a tiny ball that has wings and is enchanted. The first Snitch was a tiny bird that was very small and very tiny, but changes to the rules made it illegal to use the actual bird. The current enchanted, winged ball version of the Snitch was invented by Bowman Wright of Godric's Hollow. If the Seeker catches the Golden Snitch, their team earn 150 points and usually the win.

**Diagon Alley**

Tapping just the right brick in the wall behind the Leaky Cauldron pub in London ("Three up...two across...") will reveal an archway which is a portal into Diagon Alley, a long cobbled street where is to be found a strange and exciting assortment of shops and restaurants, some of which have tables outside with brightly colored umbrellas.