Read: H & L chapter 9.4-9.5

**DIAGON ALLEY**: The pedestrian’s map.

E is the entrance to diagon alley, and there is an exit to Charing Cross Road at R.

### Minimum Spanning Tree Problem

#### Minimum Spanning Tree Problem and Shortest Path Problem

- consider an undirected and connected network
- some measure of the positive length associated with each link
- choose a subset of links that satisfy a certain property with the shortest total length
Required property:

- **MST**: chosen links must provide a path between each pair of nodes
- **SP**: chosen links must provide a path between origin and destination

**Definition**: A spanning tree is a connected network of the \( n \) nodes that contains no undirected cycles.

**Property**: A network with \( n \) nodes requires only \((n - 1)\) links to provide a path between each pair of nodes.

**Applications**:

- Telecommunication networks
- Lightly-used transportation networks
- High-voltage electrical power transmission lines
- Wiring on electrical equipment
- Pipelines to connect a number of locations

**A greedy algorithm for the MST problem**

1. Select any node arbitrarily, then connect it to the nearest distinct node.
2. Identify the unconnected node that is closest to a connected node, add this arc to the network.
3. Ties can be broken arbitrarily. Such ties may indicate multiple optimal solutions.

**Maximum Flow Problem**

Strict upper limits have been imposed on the number of outgoing trips allowed per day in the outbound direction on each road.
The Maximum Flow Problem:

1. All flow through the directed, connected network originates from one node, called the source; and, terminates at one other node, called the sink.
2. All remaining nodes are called transshipment nodes.
3. Directed arcs indicated direction of flow, and maximum amount of flow is given by the arc capacity. At the source, all arcs point away; at the sink, all arcs point into the node.
4. Objective: maximize total amount of flow from source to sink. The amount leaving the source is equal to the amount entering the sink.

Applications:

- maximize throughput from factories to customers in distribution network
- maximize flow from vendors to factories in supply network
- maximize flow of oil through system of pipelines
- maximize flow of water through system of aqueducts
- maximize flow of vehicles through transportation network

In some applications, flow originates at more than one node and terminate at more than one node. Reformulate...

Solutions:

- Formulate as an LP and use the Simplex Method.
- Augmented Path Algorithm

Definition 1: After some flows have been assigned to the arcs, the residual network shows the remaining arc capacities (called residual capacities) for assigning additional flows.

Definition 2: An augmented path is a directed path from the source to the sink in the residual network such that every arc on this path has strictly positive residual capacity. The minimum of these residual capacities is called the residual capacity of the augmenting path because it represents the amt of flow that can feasibly be added to the entire path.
The Augmented Path Algorithm for the Maximum Flow Problem:

1. Identify an augmenting path by finding some directed path from source to sink in the residual network such that every arc on this path has strictly positive residual capacity.
2. Identify the residual capacity $c^*$ of this augmenting path by finding the minimum of the residual capacities of the arcs on this path. Increase the flow in this path by $c^*$.
3. Decrease by $c^*$ the residual capacity of each arc on this augmenting path. Increase by $c^*$ the residual capacity of each arc in the opposite direction on this augmenting path. Return to step 1.

Finding an augmenting path: Breadth-first search (fanning-out procedure).

How to know when we’re done? I.e. no more augmenting path exist.

Max-flow min-cut theorem: For any network with a single source and sink, the maximum feasible flow from the source to the sink equals the minimum cut value for all cuts of the network.

Homework 7: 9.3-1, 9.4-3, 9.5-6

Due in class October 30, 2003